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Financial Frictions in Business Cycles, Trade and Growth

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by

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To Elisa, my wife, for all these years… and to my parents, for encouraging me to continue my studies abroad.

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ABSTRACT OF THE DISSERTATION

Financial Frictions in Business Cycles, Trade and Growth

by

José Luis Wynne

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Professor David K. Levine, Chair

This dissertation is about the role of financial frictions in the business cycles, trade and growth. First, I analyze what is the role of asymmetric information problems between borrowers and lenders in the business cycles, and second, I study the role of imperfect enforcement in financial contracts on a typical country's trade and growth patterns.

In Chapter 1, I show how asymmetric information problems between banks and firms can be responsible for producing endogenous long lasting recession both at the micro and macroeconomic levels after an external shock on the interest rate faced by a small open economy. The only source of shocks is through the international interest rate or through the country risk, and the main transmission mechanism appears because banks do not observe the firms' expected productivity. In this environment, banks can infer the average quality of the firms taking each type of credit contract by observing the firms' age and net worth, thus determining credit conditions. This feature of the

model introduces heterogeneity among different generations of firms that live at the same period of time and give us insights regarding the performance of small firms along macroeconomic downturns. The results of the paper are threefold. First, unexpected increments of the interest rate produce endogenous long-lasting recessions because both the average "net worth" of the firms and their "reputation" -in financial markets- are important in generating business cycles. Second, by adding externalities in production the model is able to mimic fairly well macroeconomic and microeconomic dynamics observed along some business cycle episodes. Finally, I show that government's stabilizing policies can be welfare improving.

In Chapter 2, I study the role of financial imperfections and income distribution on trade and growth patterns. A two-sector overlapping generation economy model is analyzed where one of the sectors is characterized by an imperfection in credit markets due to moral hazard. I show that two economies with otherwise equal characteristics but with different income distribution will exhibit dissimilar comparative advantages in trade. I also analyze the dynamics of wealth distribution to show that the economy is likely to pass through different phases of trade patterns in its development process. At initial stages of development, the model economy exhibits a comparative advantage over the sector characterized by no -or less- financial frictions, to eventually revert the trade pattern at more advanced stages.

Chapter 1

Business Cycles and Firm Dynamics in Small Emerging Economies

1.1 Introduction

In the last decade we have witnessed episodes of successful and unsuccessful speculative attacks on the domestic currencies of small developing economies. Interestingly, in some of these episodes we observed that these economies entered long recessions even when the attacks were unsuccessful and con dence was recovered swiftly.[1] The

[1] See the case of Argentina 1995 below.

aftermath of these episodes was generally characterized by financial distress, especially for small firms most of which are bank-dependent.

In this work I attempt to rationalize how an unexpected and uncorrelated shock on interest rates is capable of generating a long lasting recession through an endogenous transmission mechanism. At the same time, I try to explain why small firms experience financial distress even after the cost of capital for the economy returns to normal levels.

I study a small open economy that produces tradable and non-tradeable goods where the non-tradeable good is only used as an input of production in the tradable sector. Firms in the tradable sector produce with a constant returns to scale technology and have perfect access to financial markets. Firms in the non-tradeable sector are owned by entrepreneurs who have access to a decreasing returns to scale technology where management is a fixed and indivisible factor of production. Entrepreneurs can only borrow from banks. The most important feature of this economy is the existence of an asymmetric information problem between entrepreneurs and banks about each entrepreneur's productivity. While each entrepreneur knows his own productivity, banks are unable to observe it.

At every period a constant mass of entrepreneurs is born with access to the technology to produce non-tradeable goods. Each starts up a firm and continues operating it as long as he is a successful producer. At every period in the firms' life the project undertaken by the firm can come up "successful" or "unsuccessful", where the probability of success is each entrepreneur s private characteristic. The entrepreneurs keep the same success probability over time. Whenever an entrepreneur gets an unsuccessful outcome he retires.

Because entrepreneurs know more about the quality of the investment project to be undertaken than banks do, the amount borrowed depends on the rms net worth, as casual observation suggests. The higher the net worth, the greater the ability of banks to infer that the entrepreneur has a high success probability. For this reason each rm s net worth determines its credit conditions and nancial contracts.

Also rms with a lower probability of success are more likely to default and exit, implying that the average productivity of surviving rms belonging to the same cohort improves over time.[2] Thus, the rm s age is useful observable information and nancial contracts depend on it.

It is assumed that all entrepreneurs have the same wealth at the moment of starting up their rms and that this wealth is not even close to what an entrepreneur with the highest possible productivity would need to fully nance the project by himself. For that reason, at the beginning of each cohort s life entrepreneurs need to nance a higher proportion of the rms costs by borrowing from banks. In equilibrium entrepreneurs with different productivity end up sharing the same nancial contract which turns out to be inefficient since highly productive entrepreneurs pay the same

[2] Jovanovic (1982) introduces a similar screening process of rms quality, although in his model there is no asymmetry of information since the quality is not even known by the entrepreneur who learns it over time.

cost of external finance as entrepreneurs with lower productivity. This result follows because highly productive entrepreneurs are unable signal their type to banks since they don't have enough net worth.

As time goes on successful firms build up net worth that helps highly productive entrepreneurs to separate from lower productive types. As firms are getting old, the total amount of output produced by high quality firms increases. This occurs not due to technological reasons – because firms have the same technology from birth – but due to financial ones. The banks' perception about the firms' productivity is updated each period based on age and net worth. Older and wealthier firms are perceived as better firms by banks, implying a lower cost of external finance. As older firms pay lower rates, they also produce more.

Eventually, when the highest quality firms have accumulated enough net worth the asymmetric information problem for all members of the cohort is solved, since banks are able to perfectly infer each firm's success probability. Nonetheless, it takes a long time for this to happen, and in the meantime high quality firms contract credit at a higher lending rate than the one they should be charged were information perfect. Because banks make zero profits in equilibrium, some lower quality firms contract credit at lower interest rates than they would under perfect information. This inefficiency is only fully resolved once the highest quality firms have accumulated enough wealth to truthfully signal their type.

The model is capable of producing a long-lasting endogenous transmission mechanism after a one period shock on interest rates. This happens due to two reasons. First, the speed at which information is revealed is slowed down when rms are surprised by a bad shock that reduces their net worth. Slow recovery of the rms net worth leads to a slow information revelation process since banks use this variable as a screening device in nancial contracts. In the meantime productive rms pay higher interest rates compared to steady state levels (while bad rms pay lower rates). Thus, aggregate economic performance deteriorates because of this inefficiency. I call this the net worth effect.

Second, when macroeconomic conditions deteriorate more rms might exit the industry compared to normal times. This increase in the exit rate destroys not only present but also future output since the production levels of exiting rms can only be resumed once younger generations pass through the costly screening process of building reputation over time. Again this process is costly because younger rms with high productivity are unable to convince banks to nance large investment projects since rms similar in age and equity but with low productivity have private incentives to free ride on those contracts. Hence, there is an informational loss at the aggregate level that weakens economic activity (reputation effect).

While the model with both net worth and reputation effects is able to generate strong serial output correlation after a one period shock to the interest rate, it fails to replicate the sizable economic downturns experienced in these economies. I show that by introducing an externality in production the model economy can resemble important recessions. I also show that the same externalities alone cannot explain long recessions.

The firms dynamics are also studied in this work, not only under macroeconomic steady state conditions but also along the business cycle after an unexpected shock. Time series and cross sectional information for firms drawn by simulations shows that the information revelation process is slowed down in the business cycle. This is reflected in temporarily higher lending rates, lower net worth and hence lower input-output scales of firms along the business cycle compared to steady state levels.

Finally because the sources of business fluctuations are market failures due to asymmetric information and coordination problems, there is room for policy analysis.

1.1.1 Related Literature on the Credit Channel

In the last fifteen years there has been an increasing mass of literature emphasizing the importance of asymmetric information problems in financial relationships to the credit cycle. Most of the literature focuses on the idea that it is costly for lenders to verify the output produce by ex-ante identical borrowers. Williamson (1987), Bernanke and Gertler (1989), Gertler (1992), Fuerst (1995) and its comment by Gertler (1995) and Cooley and Nam (1998) are part of this literature. The ex-ante similarity among agents and other assumptions in these models guarantees a simplifying result: there is only one optimal financial contract to solve for in the economy at each period, making models easily tractable. Yet, this simplification comes at a cost of neglecting the role of rms dynamics over the business cycles. Since all rms are equal to each other at every point in time, there can be no differential access to credit markets among them. But this result is counterfactual.[3]

The adverse selection problem modeled in this work is similar in spirit to Bernanke and Gertler (1990). There rms differ ex-ante in their success probability that is private information and net worth helps banks to imperfectly screen rms types. In my work I exploit the same idea, although it differs in two aspects. First, there is no lumpiness in investment as in their model since I allow the scale of operation to be endogenously chosen. Second, I study the adverse selection problem in a dynamic setting while theirs is essentially a static one. This change introduces a rich environment in which to study the role of rm dynamics in business cycles.

To the best of my knowledge the only work that uses heterogeneity to study the credit channel in the business cycle is Bernanke, Gertler and Gilchrist (1998). They present a model where heterogeneity is due to ex-post realizations of output. My work differs from theirs in three important respects. First, they build the nancial accelerator on top of a general equilibrium model with exogenously driven uctuations.[4] They don t attempt to make the autocorrelation observed in business cycles endogenous. Second, reputation plays no role in their credit channel story. On the contrary in my work there is value in the relationship between banks and rms because rms

[3] See Petersen and Rajan (1994) .

[4] The total factor productivity shocks are highly autocorrelated (0.95).

that have already gained good reputation will continue funding their production at low interest rates in the aftermath of the shock. Finally, in their environment the level of leverage defined as the ratio of debt to net worth and the lending rate are the same for all firms while in this model firms pay lower rates as financial reputation is acquired.[5]

Aghion, Bachetta and Banerjee (1999) study a dynamic open economy model with tradeable and non-tradeable goods where the non-tradeable good is an input of production in the tradeable. Their environment differs from the one I present in three principle aspects. First, financial frictions are built in the tradeable sector while in this paper these frictions are present in the non-tradeable one. Second, they don't study reputation and firm dynamics since they impose exogenous borrowing constraints and saving rates that are the same across firms. Third, non-tradeable output is fixed along the cycle. All the output fluctuation comes from the tradeable sector.

Lastly, Cooley and Quadrini (1998) develop a model to explain some stylized facts for US firms. Some of these stylized facts are also explained by the model economy I present. In their model they introduce moral hazard to let firms borrowing depend (proportionally) on the amount self-financed. In the present work I also have adverse selection which is eventually resolved once firms build up enough net worth. Also,

[5] Bernanke, Gertler and Gilchrist (1998) differs from Kiyotaki and Moore (1997) in that the level of leverage, although the same across firms, is endogenously determined given prices while in the latter article it is parametrically assumed.

I obtain a level of leverage (de ned as the debt-equity ratio) that is endogenously determined and dependent on the rms age. Although I loose some of the realism they get in their model, I am able to study the life cycle of rms not only in the steady state (as they do) but also along the business downturn. This is important because I believe that the fundamentals behind the rms life cycle, for example the information revelation process, contribute to the persistent poor macroeconomic performance when small rms are surprised by a bad shock.

1.1.2 The case of Argentina 1995

The impact of the Mexican crisis that took place in December 1994 on the Argentinean economy, is an example of the link between weakly correlated interest rate shocks and poor macroeconomic performance in the years that follow. The average deposit interest rate in Argentina increased in the rst quarter of 1995, and returned to its original levels right away. Yet, this short-period shock had long-lasting and profound effects on this economy, which entered in a recession that lasted almost three years as the series on the deviations from trend of the Industrial Production Index below shows.[6]

[6] Appendix A describe the series utilized here.

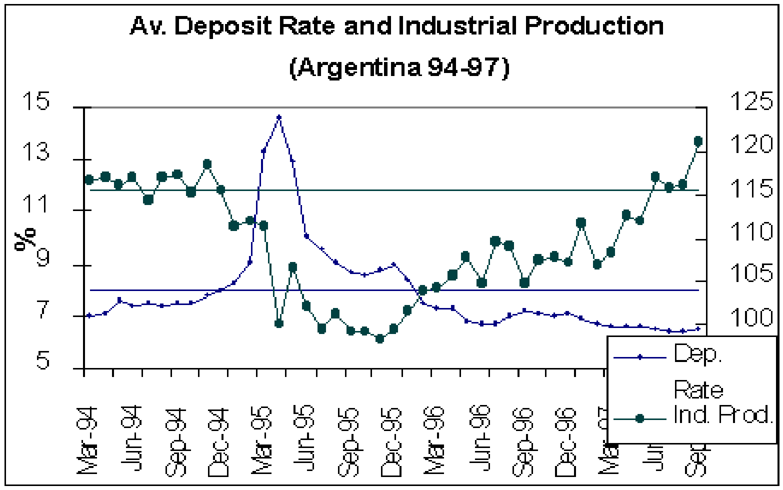

Figure 1.1: Cost of capital and aggregate output performance in Argentina 1995

This fact seems to suggest that there are strong aggregate endogenous transmission mechanisms at work, something that the standard real business cycle literature is not able to explain. While this branch of in uential literature -led by Lucas, Prescott and Kynland- and its application to small open economies -by Mendoza (1995) and Correia, Neves and Rebelo (1995)- helps us to understand the nonlinear co-movement between the main macroeconomic aggregates when exogenous perturbations occur, it is incapable of generating the autocorrelation observed in these aggregates without highly correlated shocks.

Aside from these macroeconomic facts, policy makers in Argentina have repeatedly shown concern regarding the inability of small rms to recover from an external shock because of the difficulty encountered by these rms in accessing credit in the periods that followed the shock on interest rates. The validity of this concern is evidenced by the evolution of the spread between average bank lending rates in Argentina and Libor (180 days) along the episode.

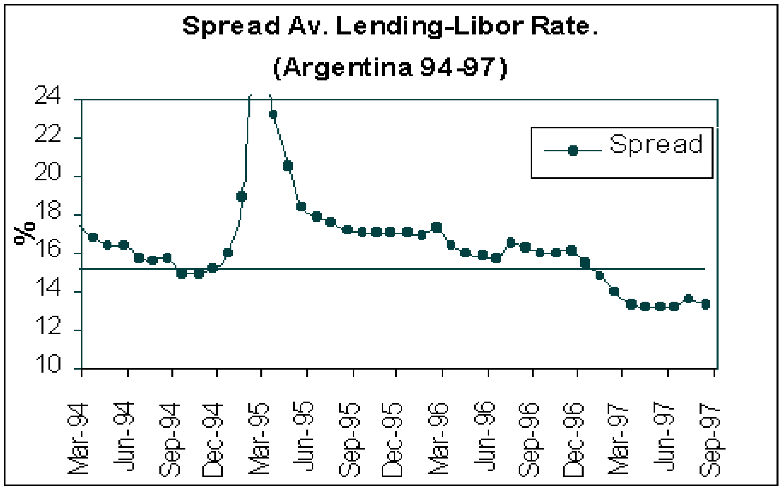

Figure 1.2: Cost of capital for bank-dependent firms.

Since the beginning of the Convertibility Plan implemented in 1991 and especially after a series of reforms to the financial system implemented in 1993, this spread continuously decreased until December 1994. The shock that occurred in the first quarter of 1995 not only sharply increased the lending rates in the first quarter of 1995, as expected, but also had a persistent effect on the spread. While these interest rates decreased in subsequent periods, the spread did not return to December 1994 levels until February 1997, more than two years later.[7]

Although the case of Argentina in 1995 is a neat example of an economy that

[7]The spread between lending rates to small firms and libor rate over the downturn of the business cycles is likely to be underestimated in this graph. In Appendix A I present evidence suggesting that small firms suffered from more severe credit constraints during this period.

enters a long recession after a temporary shock to the interest rate, one should expect the same mechanisms studied here to be present in cases where shocks exhibit more autocorrelation.

The model and the main economics problems are presented in Section 2. Aggregation issues and the description of the equilibrium are in Section 3. Section 4 discusses some analytic results. In Section 5 I present some simulation exercises and numeric results. Policy analysis is carried on in Section 6. Finally, I conclude in Section 7.

1.2 The model

There are two types of agents in the economy, workers and entrepreneurs, and three sectors, the tradable and non-tradeable goods sectors and the nancial sector. Workers and entrepreneurs consume tradeables, which are produced using capital and the non-tradable good. The non-tradable good is produced using capital and labor.

There is a mass μ of in nitely lived homogeneous workers. They are in nitely endowed with labor at every period of life and they consume only tradable goods. Their intertemporal utility function is given by:

$$U_t^W = E_t \sum_{j=t}^{\infty} (\frac{1}{r})^{j-t} \frac{\left(c_j^W - a_1 l_j^{a_2}\right)^{1-\sigma}}{1-\sigma} \qquad a_i, \sigma > 0 \tag{1.1}$$

where c_t and l_t represent consumption of tradable goods and labor supplied respectively at time t. Superscript W stands for worker. Preferences are convex and satisfy usual assumptions. Labor can be supplied at the market wage rate w_t. The discount parameter is set equal to $1/r$, where r is one plus the long run international interest rate faced by this economy. This assumption guarantees existence of a steady state equilibrium consumption path.

At each period of life, workers decide how much of their wealth to allocate in consumption and how much to save. Savings are carried via riskless assets or bonds. I assume that all assets holdings between period t and $t+1$ are represented by portfolio Γ_t expressed in consumption goods. Hence, the workers intratemporal budget constraint at every period t is given by,

$$c_t^w + \Gamma_t \leq w_t l_t + r_{t-1}\Gamma_{t-1} \qquad \forall t \geq 0 \tag{1.2}$$

where r_t is the international interest rate between period t and $t+1$.[8]

Entrepreneurs are also in nitely lived agents and consume only tradable goods. A unit mass of them is born every period and they are risk neutral agents with preferences given by

$$U_t^E = E_t \sum_{j=t}^{\infty} \gamma^{j-t} c_j^E$$

where superscript E stands for entrepreneur. Entrepreneurs have a discount factor

[8] Although the return on assets should be derived in equilibrium, I simplify notation by letting it be equal to the international interest rate from the beginning.

$\gamma < \frac{1}{r}$. Although entrepreneurs are assumed to be more impatient than workers, they will end up saving more because they have access to very pro table investment opportunities. These in nitesimal agents are endowed with one unit of labor in their rst period of life and with a project to produce non-tradeable goods in all remaining periods, contingent on having been successfully productive in the past. Entrepreneurs are assumed to be the only type of agents capable of managing inputs to produce non-tradeable goods in this economy. These goods cannot be stored. Although all entrepreneurs have the same preferences, their productivity differs. Each entrepreneurs productivity constitutes his individual characteristic, and is the second source of heterogeneity in the model, which generates some important results. To understand how this characteristic is modeled, I introduce the production technology embodied in these agents. Production of non-tradeable goods at time $t+1$ requires capital (k^N) -which is a tradable good- and labor (l) to be input at t, and it is only possible through the following technology belonging to each entrepreneur.

$$y_{t+1}^N = \theta_{t+1}(k_t^N)^\alpha (l_t^N)^\beta \qquad \alpha, \beta \ > 0, \quad i.i.d.\ \theta_{t+1} = \begin{cases} \overline{\theta} & with\ prob\ p \\ 0 & o.w. \end{cases}$$

where y_t^N stands for non-tradeable output. The random variable θ_{t+1} can take two values high, $\overline{\theta}$, or 0, and it is realized once inputs have been chosen. If the outcome of the project is unsuccessful $(\theta_{t+1} = 0)$ then the entrepreneur looses the license to produce non-tradable goods and the rms disappears.

All the parameters in this production function except for the probability p are the same across entrepreneurs. This probability constitutes each entrepreneur s characteristic and it is only observed by herself. While the parameter p is non-veri able private information, it is drawn from a publicly known density function $f(p)$ where $p\epsilon[0, 1)$. I assume that the density function is well behaved and the production function exhibits decreasing returns to scale.

Assumption 1: $\alpha + \beta < 1$.

Hence, management can be interpreted as a xed indivisible factor of production in a constant returns to scale technology. Assumption 1 imposes an upper bound on the size of the rms given equilibrium input and output prices.

I assume capital in this sector can be rented at r_k per unit of time and depreciates at a rate δ^N.

Firms exit the industry for two reasons. The rst, mentioned above, is due to unsuccessful outcomes. When entrepreneurs are unsuccessful they are unable to pay back debt. This triggers a bankruptcy process that I assume ends up destroying the rm. The second is due to reasons such as market conditions. I assumed that the exit rate due the latter argument is exogenous in this model.

Assumption 2: Entrepreneurs become unproductive with probability $q_t = \xi^{1+\chi_t}$, where $\chi_t > 0$ represents an adverse shock to the demand of non-tradeable goods and $\chi_t = 0$ implies no shock.

Thus, the probability that an entrepreneur becomes unproductive for reasons other than nancial ones depends on macroeconomic conditions. In good times this probability is just ξ, while in bad times it is assumed to be increasing on the magnitude of the shock. This parameter allows me to experiment with different exit rates.

The tradeable sector is composed of a mass of rms producing tradeable goods.[9] I assume that this sector can produce tradeables at time $t+1$ by inputting a tradeable capital good (k^T) and non-tradeable goods (y^N) at time t.[10] The technology used by this sector is given by the following generic production function,

$$y_{t+1}^T = A_t \; F(k_t^T, y_t^N) \tag{1.3}$$

where y_{t+1}^T is the rm s total output of tradeables at time $t+1$, and $F(\cdot)$ is a constant returns to scale production function, with the usual assumptions on marginal products and concavity.[11] The total factor productivity A_t is assumed to depend on aggregate non-tradeable output. This assumption allows me to study some interesting interactions between this externality and the nancial frictions built into the model. I come back to this point later. Capital utilized in this sector is assumed to depreciate at the rate δ^T.

Finally, the model is completed with the nancial sector. There is a mass of in-

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[9] For simplicity, no speci c agent operates this sector. One might assume that the sector is operated by managers that get zero payoff in equilibrium.

[10] Labor can be easily introduce as an input of production but it doesn t add any insight to the model.

[11] For simulation purposes, I assume that $F(\cdot)$ is a CES production function.

nitesimal banks, and technology in this sector is trivial. They transform one unit of tradable goods borrowed into one unit of tradable good lent at no cost (xed or marginal). They raise funds by issuing debt (deposits) to workers and other international investors, and they lend those funds to small entrepreneurs.[12] This sector is introduced to keep the economy decentralized and to make clear assumptions on debt contracts.

Assumption 3: Banks observe each rms age and net worth only.

Banks do not observe the entrepreneur s characteristic. They only observe the type of contract that their clients are taking. Since in equilibrium there are separating contracts, the banks can infer what is the exact productivity of the client when these separating contracts are taken. I assume that banks don t observe contracts that rms signed with other banks.

Assumption 4: Only one period debt contracts are enforceable.

This assumption rules out the possibility that banks offer contracts where they get to keep all the rms revenues for a certain number of periods before nally letting the surviving rms recover control. I assume that this kind of contract is too costly to implement since banks are unable to monitor the entrepreneur s behavior. Simple moral hazard problems where entrepreneurs can abscond with part of their proceeds make multiperiod nancial contracts unfeasible. Assumption 4 has the purpose of

[12] Banks are owned by foreign agents.

ruling these alternative contracts out without having to complicate the environment.

Given Assumption 4, banks are allowed to commit to offer any one period debt contract they want in the future. I come back to this point later when I solve for the equilibrium contracts. Also, I assume that rms are unable to commit to future production plans.

In the rest of the paper I analyze the limiting case where the probability of having an adverse shock to the interest rate goes to zero. Then I hit the economy with a one period shock. This case study allows for tractability while still giving insights regarding the transmission mechanisms that work along business cycles downturns in these economies, which is the principal focus of the paper.

In the next subsections I present the tradeable, nancial and non-tradeable sectors problems.

1.2.1 The tradeable sector s problem

The objective in this sector is to maximize intertemporal pro ts. Thus, the problem at each period of time is

$$\begin{aligned} \max_{\{y_t^N, k_t^T, x_t\}_{t=0}^{\infty}} \pi_t^T &= \sum_{j=t}^{\infty} \left(\prod_{i=0}^{j-t} \frac{1}{r_i} \right) \left[AF(k_{j-1}^T, y_{j-1}^N) - P_j^N y_j^N - x_j \right] && (1.4) \\ s.t. \quad k_t^T &= (1-\delta^T) k_{t-1}^T + x_t \quad ; \quad x_t > 0 && (1.5) \end{aligned}$$

$$y_{t-1}^N, k_{t-1}^T, \{P_t^N, r_t\}_{t=0}^{\infty} \quad given.$$

where x_t denotes the rm s investment level. At any period t, and given the timing of production, total tradable output has already been chosen. There is no uncertainty for this sector since the interest rate at t is known when inputs are decided. Also note that investment becomes capital or productive right away. Finally, investment in the sector is irreversible.

The rst order conditions of this problem are

$$AF_{y^N}(k_t^T, y_t^N) = P_t^N r_t \tag{1.6}$$

$$AF_k(k_t^T, y_t^N) = (r_t - 1 + \delta^T) \tag{1.7}$$

Both conditions implies that the value of the marginal product of both inputs should equal their marginal cost at the optimum.

Now we turn to the non-tradeable rm s problem.

1.2.2 The entrepreneurs problem

The entrepreneur s problem is more complex due the asymmetry of information between them and the rest of the agents in this economy. As was mentioned before, only small rm owners have the technology to produce non-tradeable and each of them is embodied with a privately known probability p of having a high output performance. Because of this heterogeneity and the fact that entrepreneurs keep their characteristic through time if they have successfully produced in the past, not all problems for different owners will be the same. The setup of the problem will differ across entrepreneurs characteristics and ages, since useful information is revealed over time.[13] For this reason I denote with subscripts nt an entrepreneur of age n at time t.

An entrepreneur s rst-period problem is trivial: he supplies all his labor endowment and save all his income. The problem becomes less trivial for subsequent periods. In all these periods, a small rm owner decides how to allocate his wealth NW_{nt} between consumption c_{nt}^E and savings. He can save by investing some of the savings in his small rm (e_{nt}) and/or by investing in safe assets at the international interest rate. Nonetheless, an entrepreneur never saves in safe assets given assumptions on preferences and the subjective discount rate.

Investment within the rm is allocated between capital (k_{nt}^N) and labor (l_{nt}^N) to produce non-tradeable goods, given input prices, expected output prices P_{t+1}^N, entrepreneurs wealth and available nancial contracts.

The assumptions made in the model restrain the nancial agreements to simple debt contracts. These contracts will depend on the rm s net worth and its age but not on its owner characteristic since it is non-observable. The contract is a duple $\{M_{nt}(e_{nt}), i_{nt}(e_{nt})\}$, where M_{nt} stands for the size of the loan and i_{nt} for one plus the lending interest rate charged to an entrepreneur of age n at date t.[14] The contract is

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[13] Note the non-recursive structure of each entrepreneur s problem.

[14] To simplify notation I assume that a sufficient contract only speci es net worth and age, but

a function of the rm s net worth because this variable is one of the bank screening devices to imperfectly infer the entrepreneur s characteristic. This point will become clearer once I set up the nancial sector s problem and show how to solve for the equilibrium of the model.

Before specifying the entrepreneur s problem I present the maximization problem that allows to compute the rm s return.

Because the entrepreneur s discount rate is higher than the interest rate by assumption, he will always borrow from the bank as long as he is productively successful. The gross expected return on investment e_{nt} under external nance per period is denoted as $TR_{nt}(e_{nt}, p)$. Taking contracts as given, this return function comes from the following problem

$$\max_{\{k_{nt}^N, l_{nt}\}} TR_{nt}(e_{nt}, p) = p[P_{t+1}^N \ \bar{\theta} \ (k_{nt}^N)^{\alpha}(l_{nt}^N)^{\beta} - i_{nt}(e_{nt})M_{nt}(e_{nt})] \tag{1.8}$$

subject to

$$r_{k_t} k_{nt}^N + w_t l_{nt}^N \leq e_{nt} + M_{nt}(e_{nt}) \tag{1.9}$$

Thus, the entrepreneur s expected return of investing e_{nt} in her small rm having characteristic p, is given by total output in case of good productive performance minus the amount due next period, the loan s principal plus interest. Equation (9) is a

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the reader should keep in mind that contracts are also over production plans.

budget constraint: total cost of investment has to be nanced with internal or external funds, where the external funding comes only from the bank. It is worth noting that maximizing this one period return for the rm will result in a maximization of the entrepreneurs utility as long as the sequence of net worth chosen is optimal.

Having described how returns are computed, and letting τ be the number of periods that have passed since the entrepreneur was born, I next set up the entrepreneur s problem assuming it starts once labor has been supplied. It is at this stage of the problem that total wealth is optimally divided between present consumption and savings via the rm s net worth. Thus,

$$\max_{\{c^E_{nt}, e_{nt}\}} U^E_{nt} = E_t \sum_{j=t}^{\infty} \gamma^{j-t} c^E_{(j-\tau)j} \tag{1.10}$$

subject to

$$\begin{aligned} c^E_{nt} + e_{nt} &\leq NW_{nt} \quad \forall nt. \\ NW_{n't'} &= \begin{cases} w_t & for \quad n = 1 \\ TR_{nt}(e_{nt}, p) & \forall n > 1 \end{cases} \end{aligned} \tag{1.11}$$

where subscript $n't'$ denotes the entrepreneur s decision variables at $t+1$.

Given the assumptions in the model, the rm s total revenue function at all times is differentiable with respect to e_{nt}. Thus, the rst order condition with respect to

e_{nt} (and consumption) can be computed for all periods and ages.

$$1 \leq \gamma \frac{\partial E_t \left[TR_{nt}(e_{nt,} p) \right]}{\partial e_{nt}} \qquad if < 0, \; then \; e_{nt} = NW_{nt} \tag{1.12}$$

which means that all the entrepreneur s wealth should be allocated to the rm if marginal returns there are higher than $1/\gamma$. If this condition holds with equality, an interior consumption solution arises. For this we need to solve for the return function $TR_{nt}(e_{nt,} p)$, which I do after de ning equilibrium.

Next, I complete the productive structure of this economy with the nancial sector.

1.2.3 The nancial sector s problem

As mentioned above, this sector is composed of in nitesimal nancial institutions offering standard debt contracts to entrepreneurs and raising funds at the international interest rate from workers or foreign investors through bank deposits. As is usual, banks participate if they make non-negative expected pro ts.

Banks choice variables are the size of the loan and the lending interest rate under all types of contracts. Thus the bank s objective function, assuming there is some demand for loans, will be.

$$\max_{\{M_{nt}, i_{nt}\}} E\pi^F_{t+1} = \overline{p}_{nt}(e_{nt}) i_{nt} M_{nt} - r M_{nt} \tag{1.13}$$

subject to

$$\overline{p}_{nt}(e_{nt}) = E_t \left[p \mid p >\in PC(e_{nt}, i_{nt}, M_{nt}),\ \ f_{nt}(p) \right] \tag{1.14}$$

where $\overline{p}_{nt}$ is the average quality of a rm of age n at t engaging in this credit contract. Note that this average is computed as the average entrepreneur s quality of those who are willing to participate in the contract $\{i_{nt}, M_{nt}\}$ and who have the same net worth e_{nt}, given all other alternative nancial contracts. The mass of entrepreneurs of age n at time t with characteristic p comes from a known density function $f_{nt}(p)$.

It is useful to see for future reference that: a) if only one type of entrepreneur is willing to participate in a contract, then the average quality is given by that type, and b) if all types $p > p^*_{nt}$ are willing to participate, the average type can also be computed. While the bank is unable to observe individuals characteristics, it knows $f_{nt}(p)$ and it is able to compute the lowest quality type that will participate in the contract. This density function can be computed using the density function of rms of age n alive at every period t. Assuming that there was no bad aggregate shock in the history of these rms, this function is $f(p)(\xi p)^{n-1}$. In other words, it is density function of rms that were born together conditional on being alive n periods later. Thus, the density function of those alive in their rst period of life ($n = 1$) is just the density function of the newborns. Moreover, because banks can inferred the average type taking each contract at every period, it also knows the average type that have taken contracts in previous periods.

1.2.4 The worker s problem

Workers are passive players in this model. As mentioned before they supply labor and buy and sell assets to maximize intertemporal utility. Since the mass of rms in the nancial and tradable sectors is the same as the mass of workers, I let all rms to be owned by workers. This reduces accountability problems and simpli es notation without changing any results. Then, the consumers solve the following problem.

$$\max_{\{c_t^W, l_t, \Gamma_t\}_0^\infty} U_t^W = E_t \sum_{j=t}^{\infty} (\frac{1}{r})^{j-t} \frac{\left(c_j^W - a_1 l_j^{a_2}\right)^{1-\sigma}}{1-\sigma} \qquad a_i, \sigma > 0 \tag{1.15}$$

subject to

$$c_t^w + \Gamma_t \leq w_t l_t + r_{t-1}\Gamma_{t-1} \qquad \forall t \geq 0 \tag{1.16}$$

$$\Gamma_0 \textit{ and } \{w_t, r_{t-1}\}_{t=0}^\infty \textit{ given.}$$

$$\lim_{t\to\infty} \frac{\Gamma_t}{\Pi_{\tau=0}^t r_\tau^\tau} \geq 0 \tag{1.17}$$

Equation (17) rules out *Ponzi* schemes. The rst order conditions for this problem in the limiting case where the probability of the shock goes to zero give us:

$$c_t^W - a_1 l_t^{a_2} = \left(\frac{r}{r_t}\right)^{1/\sigma} \left(c_{t+1}^W - a_1 l_{t+1}^{a_2}\right) \qquad \forall t > 0 \tag{1.18}$$

$$l_t = \left(\frac{w_t}{a_1 a_2}\right)^{\frac{1}{a_2 - 1}} \qquad \forall t > 0 \tag{1.19}$$

$$\sum_{t=0}^{\infty} \left[c_t^w + \Gamma_{t+1} - w_t l_t - r_{t-1}\Gamma_t\right] \leq 0 \tag{1.20}$$

and the transversality conditions for assets.

Equation (18) is the law of motion for consumption and Equation (19) is the labor supply in the non-tradeable sector. From the assumptions on preferences we are able to derive a labor supply that is independent of present or future consumption and therefore independent of income. This is important to compute the equilibrium transition from one steady state to the other, after the economy is perturbed by an exogenous shock.

Note that with these preferences, workers try to smooth $\left(c_t^W - a_1 l_t^{a_2}\right)$ but not consumption. Having completed the description of the workers problem, I closed the model with aggregation details to nally de ne equilibrium for this economy.

1.3 De nition of Equilibrium

To avoid postponing the de nition of equilibria, I present a heuristic description over the types of equilibrium contracts that arise in this economy.[15] There are two types of equilibrium contracts, and they are

De nition 1 *A pooling nancial contract* $\{i_{nt}^{Pool}(e), M_{nt}^{Pool}(e)\}$ *is a simple debt contract in which more than one type of entrepreneurs participate.*[16]

De nition 2 *A separating nancial contract* $\{i_{nt}^{Sep}(e(\hat{p})), M_{nt}^{Sep}(e(\hat{p}))\}$ *is a simple debt contract in which only those entrepreneurs that truthfully reveal the same type participate, where* $\hat{p}$ *is the announcement of each entrepreneur s type.*

In fact, those entrepreneurs that belong to the same cohort and with characteristic $p > p_{nt}^{*}$ will participate in the same pooling contract, sharing the same production plan and the same ex-post output (although the probability of getting a high output will differ across those with different p). The average quality type participating in a pooling contract is then

$$\overline{p}_{nt} = E_t(p/p > p_{nt}^{*}, f^n(p)) = \frac{\int_{p_{nt}^{*}}^{1} f(p)\ p^n dp}{\int_{p_{nt}^{*}}^{1} f(p)\ p^{n-1} dp} \qquad with \;\; p_{nt}^{*} \in [p_{(n-1)(t-1)}^{*}, 1] \quad (1.21)$$

where p_{nt}^{*} is the lowest quality type taking a pooling contract at time t in a cohort of

[15] A formal proof is presented in the next section.

[16] *Note that the pooling contract does not depend on the characteristic parameter.*

age n.[17]

Entrepreneur that belong to this cohort with characteristic $p < p^*$ will be engaged in truth telling ($\hat{p} = p$) separating nancial contracts from then on. As mentioned before this contracts are a function of the rms age and net worth only.

The model is closed by specifying the mass of agents of each type. As mentioned before, at each moment in time there is a mass μ of rms producing tradeable goods, banks and workers. Computing the mass of entrepreneurs is not a trivial task due to the heterogeneity of the model. To de ne equilibria we need to know for each cohort the mass of rms of the same age taking a truth telling contract (those that have characteristic parameter lower than p^*_{nt}) and the mass corresponding to those from the same cohort taking a pooling contract (with characteristic parameter higher than p^*_{nt}). This is because allocation of labor and capital inputs depend on the amount nanced to each type.

Before computing this, note that the mass of entrepreneurs productively active at each moment in time is the sum of those that are one, two and so periods old. In the absence of an aggregate bad shock history $q_t = q \forall t$, this total mass can be computed in the following way.

[17] Note that $p^*_{nt} \in [p^*_{(n-1)(t-1)}, 1]$, where $p^*_{(n-1)(t-1)}$ is a state information variable of the model, which is known by all banks.

$$
\begin{aligned}
M_t^E &= \int\limits_{p\epsilon[0,1)} [f(p) + qpf(p) + (qp)^2 f(p) + ...]dp \\
&= \int\limits_{p\epsilon[0,1)} [\frac{f(p)}{1-qp}]dp \qquad (1.22)
\end{aligned}
$$

where M_t^E is the mass of rms at each point in time. This mass is nite and independent of time as long as there is no history of aggregate shock.

Also we are able to distinguish the total mass of rms under a pooling contract and the total mass under a truth telling or separating contracts.

Variables p_{nt}^*de ne the threshold for each cohort n at date t that separates those rms taking truth telling contracts from those still in a pooling contract. Thus, for a cohort n at time t, a fraction

$$\eta_{nt} = \int\limits_{P_{nt}^*}^{1} (q_t p)^n \ f(p)dp$$

will take the pooling contract and a fraction

$$\int\limits_{0}^{P_{nt}^*} (q_t p)^n \ f(p)dp$$

will take a truth telling contract. This is true for all cohorts. Note that if p_{nt}^* reaches a value one for some cohort at some time, everybody in this group will take truth telling contracts and all asymmetry of information is solved among them from then on. I come back to this point later.

Because this is a small open economy model, equilibrium is determined by emptying the labor and the non-tradeable good markets and requiring intertemporal resource constraints for workers to be satis ed.

Let $(\mu, \{(\eta_{nt})_{n=1}^{\infty}\}_{t=1}^{\infty}, f(p))$ be the economy described above, where $\{(\eta_{nt})_{n=1}^{\infty}\}_{t=1}^{\infty}$ determines the mass of all rms alive at t that were born at $t-n$ and come from a density function $f(p)$ which by assumption is constant over time.

De nition 3 *A competitive equilibrium for economy* $(\mu, \{(\eta_{nt})_{n=1}^{\infty}\}_{t=1}^{\infty}, f(p))$ *is a collection of state variables* $\{[\eta_{nt}, p^{*}_{(n-1)(t-1)}, NW_{nt}(p)_{p=0}^{p^{*}_{nt}}]_{n=1}^{\infty}\}_{t=0}^{\infty}$, *a collection of inputs, nancial contracts and output for the entrepreneurs taking a pooling contract,* $\{(k^{N}_{nt}, l_{nt}, i^{Pool}_{nt}(e), M^{Pool}_{nt}(e), y^{N}_{n+1t+1})_{n=1}^{\infty}\}_{t=0}^{\infty}$, *a collection of inputs, nancial contracts and output for all entrepreneurs taking separating contracts,* $\{(k^{N}_{nt}(p), l_{nt}(p), i^{Sep}_{nt}(e(\hat{p})), M^{Sep}_{nt}(e(\hat{p})), y^{N}_{n+1}(p))_{n=1}^{\infty}\}_{t=0}^{\infty}$, *inputs and output for the tradable sector,* $\{Y^{N}_{t}, K^{T}_{t}, Y^{T}_{t+1}\}_{t=0}^{\infty}$[18], *all entrepreneurs consumption allocations* $\{(c^{E_p}_{nt})_{n=1}^{\infty}\}_{t=0}^{\infty}$, *workers consumption allocation, labor supplied and portfolios* $\{c^{w}_{t}, l_t, \Gamma_t\}_{t=0}^{\infty}$ *and prices* $\{r_t, w_t, P^{N}_{t}\}_{t=0}^{\infty}$ *such that,*

- $\{(k^{N}_{nt}, l_{nt}, i^{Pool}_{nt}(NW_{nt}), M^{Pool}_{nt}(NW_{nt}), y^{N}_{n+1})_{n=1}^{\infty}\}_{t=0}^{\infty}$ *is the solution to all entrepreneurs problems of age* n *at time* t *with parameter* $p \geq p^{*}_{nt}$ *and net worth* NW_{nt}.

[18] *A competitive equilibrium can be solved assuming there is only one rm producing tradable goods.*

- $\{(k^N_{nt}(p), l_{nt}(p), i^{Sep}_{nt}(e(p)), M^{Sep}_{nt}(e(p)), y^N_{n+1}(p))^{\infty}_{n=1}\}^{\infty}_{t=0}$ *is the solution to all entrepreneurs problems for all owners of rms of age n at time t with parameter $p < p^*_{nt}$ and wealth $NW_{nt}(p)$.*

- *Pooling and separating contracts solve the Banks problem.*

- $\{Y^N_t, K^T_t, Y^T_t\}^{\infty}_{t=0}$ *is the solution to the tradable sector s problem,*

- $\{(c^{Ep}_{nt})^{\infty}_{n=1}\}^{\infty}_{t=0}$ *are the consumption allocations of entrepreneurs of type p and age n at every period t.*

- $\{c^w_t, l_t, \Gamma_t\}^{\infty}_{t=0}$ *is the solution to the workers problem. Finally,*

- *Markets clear:*

Equilibrium in the labor markets

$$\sum_{n=1}^{\infty}\left[\eta_{nt}\ l_{nt} + \int_0^{P^*_{nt}} l_{nt}(p)\ (q_t p)\ f(p)dp\right] = b + \mu l_t \qquad \forall t \geq 0.$$

or aggregate labor demand (demand across rms by type and mass) equal labor supply.

Equilibrium in the non-tradeable market or

$$\sum_{n=1}^{\infty}\eta_{nt}\left[\int_{P^*_{nt}}^{1} p\ y^N_{n+1t+1}\ (q_t p)^n\ f(p)dp + \int_0^{P^*_{nt}} p\ y^N_{t+1}(p)\ (q_t p)^n\ f(p)dp\right] = Y^N_t \quad \forall t \geq 0.$$

Note that labor demand is the sum of labor demanded by rms under pooling contracts (all of them having the same production plan), plus labor demanded by rms under separating contracts (each having different production plans). Also note that total non-tradeable output is computed following the same reasoning, although the total output produced under a pooling plan will be the expected output knowing that each entrepreneur with characteristic p in the same pooling contract will produce an average non-tradeable output of $p\ y_{n+1}^{N}$. Because this happens for all types in the pool, aggregation is given by expression $y_{n+1}^{N} \int_{P_{nt}^{*}}^{1} p\ (q_t p)^n\ f(p)dp$.

1.4 Analysis of the model

To prove existence of equilibrium I present some analytic results that are also useful to get some insights of the model s predictions. Due to the huge source of heterogeneity the reader might think that the problem cannot be solved. Nonetheless, the model is solvable not only for the steady state but also out of it.

In the next subsection I work under the assumption that all prices in the economy are constant over time, and I show that the types of contracts described above are actually equilibrium contracts. I explain how to solve for separating and pooling contracts for members of the same cohort. This implies determining which entrepreneurs types of the same cohort end up with a pooling contract and which with separating ones.

Then I show how contracts are allocated between members of the same cohort in successive periods. This is useful to observe how inefficiencies vanish over time in the same cohort. In other words, I show that over time, the set of entrepreneurs types taking a pooling contract shrinks, meaning that more and more types will take a contract that only ts themselves and that the asymmetry of information is eventually resolved in the cohort. In this process we observe how banks learn the rms productivity as these entrepreneurs build up net worth .

Later I show how the shock to the interest rate affects the price of non-tradable goods, surprising rms in this sector.

Finally I explain that both types of nancial contracts mentioned before are also equilibrium contracts (with some minor changes) after the economy is perturbed by the shock to the interest rate. It is at this stage where the assumption that banks learn their clients type by observing the type of contracts they took in the past comes into play. Since some information about an entrepreneurs types has been revealed (since they have some reputation) banks will make use of this information after the shock, even if the net worth of the rms (that helps to signal entrepreneurs types to banks) is drastically reduced. Although the economy doesn t loose information already acquired, the information revelation process is slowed down after a bad shock since rms loose net worth . Because of the fact that information is never destroyed I refer to this information revelation process as reputation acquisition. Once rms get to build up some reputation, they will keep it as long as they are productively successful.

This efficient use of the information explains the value of the lender-borrower relationships analyzed by Petersen and Rajan (1995) and Petersen and Rajan (1994). Because it is important to have a relationship with a bank, there might be incentives for rms to keep borrowing from the same intermediary throughout time.

The reputation acquisition feature of the model challenges previous work on the credit channel where reputation is absent. Because the loss of information in such a world are overstated, the role of the credit channel as a propagation mechanism might be overrated. In the present work, I show that although reputation reduces the damage in the economy when shocks arise, weakening the net worth effect stressed in the literature, it also introduces the feature that it takes a long time for rms to build up reputation. Thus, if some rms die along the downturn of the business cycle, it takes a long time for the economy to replace them.

1.4.1 Financial contracts in steady state

Assume we are looking at a newborn cohort of entrepreneurs that are just starting up their rms after having supply their labor endowment, which by assumption happens only once. For the time being assume that prices are at steady state levels.

Figure 1 shows the mass of these newborn entrepreneurs that belong to the same cohort under the assumption, as in the simulation exercises followed later, that $f(p) =$

$6p(1-p)$.[19]

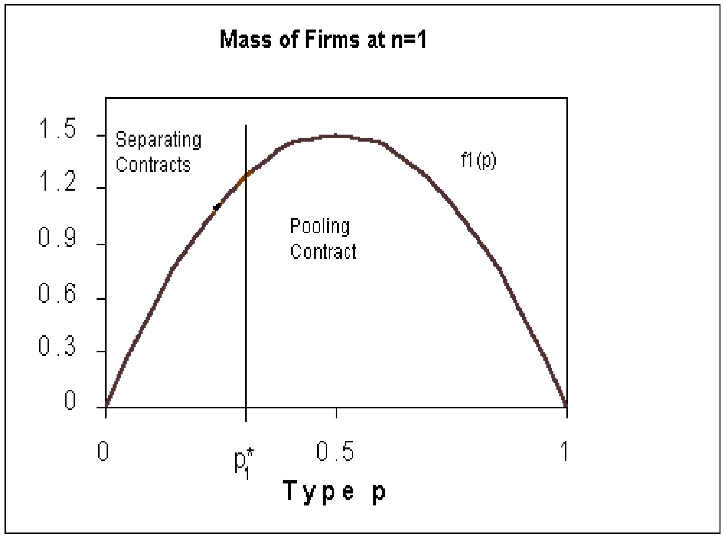


Figure 1.3: Contract characterization for newborn entrepreneurs

The approach followed here is to show that the contracts described before as well as the inducted allocations actually constitute an equilibrium. To prove this I show that all agents maximize their expected utility given market prices. Entrepreneurs are assumed to take contracts as given. Banks can come up with new contracts if the ones proposed by other banks in the market are not equilibrium ones.

As I mentioned before, there are two types of equilibrium contracts, separating and pooling contracts. The former ones have the characteristic that each type will get a different contract while in the latter ones more than one type participates.

The equilibrium contracts for a 1 year old cohort are such that all types in this cohort with $p < p_1^*$ those with success probability below some threshold p_1^* will

[19]More general density functions do not change the result as long as $f(p) > 0$ for all $p \in [0, 1]$.

take separating contracts (see Figure 1). In other words, they will take a nancial contract that no other type will be willing to take. Also all types with age 1 and with $p \geq p_1^*$ will share the same pooling nancial contract. In this section I explain why these are equilibrium contracts and where the p^* threshold is coming from. I Also explain that this threshold is increasing over time until it reaches the upper bound of the distribution of types. It is then when the asymmetry of information in the cohort is resolved.

Figure 2 shows how the mass of entrepreneurs changes over time due to the fact that unsuccessful entrepreneurs disappear. If the types of newborn rms is given by the density function assumed above, when the cohort is n periods old the density function is $f_n(p) = 6p^n(1-p)$.

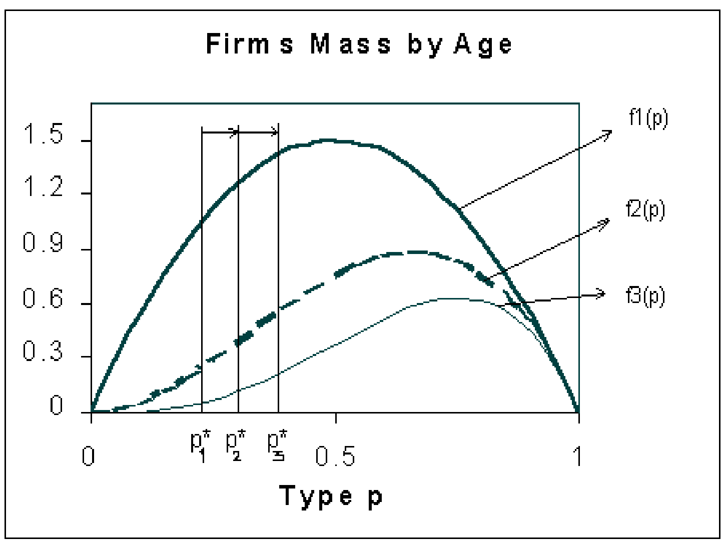


Figure 1.4: Contract characterization by type and age for all entrepreneurs

The thresholds p_n^* show the cut off points between types taking separating and pooling contracts over time. In steady state p_n^* is a non-decreasing function of n, the cohort s age. In the picture p_n^* is a strictly increasing function of n because of assumptions on $f_1(p)$. This help to facilitate computations although it doesn t alter the results. I ll discuss this point in detail later.

Letting the threshold be also indexed by time, the rst result obtained is given by the following proposition.

Proposition 1 *The average quality rm in a pooling contract, $\overline{p}_{nt}^{pool}$, is an increasing function of both p_{nt}^* and n.*

Proof. See Appendix.

The average quality increases with the age of the cohort (holding the lowest type participating in the pooling contract constant) because as time passes lower types die with higher probability as the reader can see from Figure 2. For the same cohort this average also increases with p_{nt}^* as lower types exit pooling contracts.

To understand how contracts work, let s focus on the equilibrium along a steady state path. For this reason I drop time subscript on prices for the purpose of this subsection.

By Assumption 4 banks cannot commit to multiperiod nancial contracts, although they can commit to offer any kind of one period nancial contracts in the future. An equilibrium contract is a pair composed by a lending rate and a loan, $\{i_{nt}(e), M_{nt}(e)\}$, specifying age and net worth e (which is observable information), such that rms maximize pro ts subject to: 1) technological constraints, 2) available nancial contracts and 3) banks participation constraint.

These contracts are solved using a principal agent approach. As was explained in Section 2, the entrepreneur s problem can be divided in two steps. First, we solve for the returns of the rm in a period by period basis as a function of the rm s net worth and then we solve for the optimal allocation of the entrepreneurs wealth between consumption and investment in the rm. After having done the last step, we can go back to check whether the nancial contract that comes out of the rst step is actually consistent with the equilibrium conditions de ned in Section 3.

An entrepreneur with characteristic p (assumed to be high enough) and internal funds e_{nt} can compute his own return function $TR_{nt}(e_{nt}, p)$ by solving the following problem[20]

$$\max_{\{k_{nt}^N, l_{nt}^N, i_{nt}, M_{nt}, \bar{p}, p^*\}} E_t[TR_{nt}(e_{nt}, p)] = p[P^N\ \bar{\theta}\ (k_{nt}^N)^\alpha (l_{nt}^N)^\beta - i_{nt}M_{nt}] \tag{1.23}$$

subject to

$$\bar{p}_{nt}\ i_{nt}M_{nt} - rM_{nt} \geq 0 \tag{1.24}$$

$$r_k k_{nt}^N + w l_{nt}^N = e_{nt} + M_{nt} \tag{1.25}$$

[20] The problem looks the same whether the economy is in steady state or not, though it simpli es notation to assume it is, since time subscripts for all prices can be dropped.

$$\overline{p}_{nt} = E_t[p \; | p \in PC(e_{nt}, i_{nt}, M_{nt}), \;\; f_{nt}(p)] \tag{1.26}$$

Note that all the microeconomic variables especially the nancial contract depends on the age of the rm because it is observable. The choice variables are inputs (i.e. capital and labor), nancial contracts (i.e. principal plus interest) and the average quality rm participating in the contract. Implicitly we need to nd the lower quality type in the contract. The lower quality type in pooling contract is p^*_{nt}. In separating contracts this is just p, since there is only one type taking it. Finally the objective function is the expected return for the rm with success probability p. It is interesting to notice that the problem for rms with different characteristics that take the same (pooling) contract looks the same except for the fact that the objective function of one type is a positive transformation of the others. This feature will facilitate aggregation across types taking the same contract.

Equation (24) is the bank s participation constraint. Total expected return on loans should be at least equal to the cost of funds (given by the international interest rate). Equation (25) is a budget constraint: total cost of production must be nanced with either internal funds or loans. Equation (26) de nes the average quality, which is computed by averaging across the types $p \in PC$ in the same nancial contract, and knowing $f_{nt}(p)$, the density function of the rms of age n at time t. The types $p \in PC$ are determined in equilibrium.

Before solving the problem under asymmetric information, it is worth noting that in a fully informed environment there is no free riding since financial agreements would internalize the default probability by raising the lending rate of the contract as in Modigliani and Miller (1958). Thus,

Proposition 2 *: (Modigliani and Miller's Neutrality Theorem). Under complete information, the optimal amount of labor and capital hired to produce non-tradeable goods is independent of the firms' wealth .*

Proof. *See Appendix A.*

The basic intuition behind this theorem is that if the entrepreneur and banks have the same information regarding the success probability of the firm, then there is no conflict of interests among them and they will work out a financial contract such that the efficient scale of production is implemented. In this world of full information, shocks to the entrepreneurs' net worth do not change the aggregate production level. Moreover, firms do not grow over time since they start up right away at the efficient level of production.

In a world with asymmetric information matters are different. The asymmetry of information opens interesting dynamics at the firm level that impact on the macroeconomy both at the steady state and along the downturn of the business cycles.

In this case, an analytic solution for financial contracts and inputs is not possible. Nonetheless, the optimal level of capital and labor can be solved as a function of

$\bar{p_{nt}}$, the average quality type in the same nancial contract (which is an endogenous variable of the problem).

Proposition 3 *Solutions for inputs under one-period debt contracts are given by*

$$k_{nt}^N = \left[\frac{\bar{p_{nt}}\ P^N\ \bar{\theta}\ \alpha^{1-\beta}\beta^{\beta}}{w^{\beta}\ r\ r_k^{1-\beta}} \right]^{\frac{1}{1-\alpha-\beta}} \tag{1.27}$$

$$l_{nt}^N = \left[\frac{\bar{p_{nt}}\ P^N\ \bar{\theta}\ \alpha^{\alpha}\beta^{1-\alpha}}{w^{1-\alpha}\ r\ r_k^{\alpha}} \right]^{\frac{1}{1-\alpha-\beta}} \tag{1.28}$$

Proof. *See Appendix.*

The variable $\bar{p_{nt}}$ can be interpreted as the banks perception about the average quality rm taking the contract. Note that Proposition 6 also holds for a truth telling separating contract (by letting $\bar{p_{nt}}= p$).

Inputs depend negatively on their prices and positively on the price of the nal good and the productivity parameter $\bar{\theta}$. More meaningfully, both inputs depend positively on the average quality of the pool since the loan interest rate depends on it. A better average reduces the interest rate on loans and increases the demand for both inputs. It is interesting to notice that the actual productivity doesn t appear in Equation (27) and (28). Thus, the total output is determined only by the bank s perception about the rms average productivity ($\bar{p}$). This occurs because banks are the marginal suppliers of funds when entrepreneurs do not have access to other nancial sources.

Using Equation (27) and (28) we can collapse the entrepreneur s problem even further. Now total return for rms becomes

$$\max_{\{k_{nt}^N, l_{nt}^N, \bar{p}, p^*\}} E_t[TR_{nt}(e_{nt}, p)] = p\left[(1-\alpha-\beta)\left(\frac{\bar{p}^{\alpha+\beta}\ P^N\ \bar{\theta}\ \alpha^\alpha \beta^\beta}{w^\beta\ r^{\alpha+\beta}\ r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}} + \frac{r}{\bar{p}} e_{nt}\right] \tag{1.29}$$

subject to

$$\overline{p}_{nt} = E_t[p\ |p \in PC(e_{nt}, i_{nt}, M_{nt})\ ,\ f_{nt}(p)] \tag{1.30}$$

The expected return on e_{nt} is increasing in the average quality of the rm for low net worth levels. It will be shown later that in equilibrium the return is always increasing in the average quality.

Next, I address the question of whether it is possible for banks to offer (non-linear) nancial contracts such that every entrepreneur taking a contract would be willing to truthfully reveal his own type. These contracts exist under two conditions. First, the level of net worth invested in the rm within the period has to be big enough to make the entrepreneur s type announcement credible. Note that in the extreme case where entrepreneurs nance all the cost of production, they have no incentives to lie. In equilibrium, banks will lend to rms since entrepreneurs have a subjective discount factor that is bigger than the interest rate. By making nancial contracts where the amount self nance (net worth) is increasing on the announcement, banks can make sure that all types reveal truthfully. Thus any intermediate type faces a trade off:

announce a higher type, invest more and pay lower borrowing rates if successful, or announce his own type and invest a lower amount which lead him to consume the difference sooner for sure. Note that entrepreneurs with low probability of success behave as if they were more impacient.

Second, all future contracts have to be as demanding as the rst truth telling contract in terms of the amount nanced internally. Otherwise, some entrepreneurs may imitate others for a number of periods knowing that they can free ride on these others future contracts. This condition is satis ed since banks can commit to offer the same type of contracts in the future. Then, if there is no gain from free riding in the present, there is no gain from doing it in the future because contracts are expected to be the same over time.

Let p be the rm s true characteristic and $\hat{p}$ its announcement. A truth telling contract is $\{i_{nt}(e(\hat{p})), M_{nt}(e(\hat{p}))\}$, where the entrepreneur has incentives to announce $\hat{p}= p$.

Proposition 4 *A truth telling contract is given by*

$$\hat{p}= p$$

$$e(p) = \frac{(1-\alpha-\beta)\gamma r(\alpha+\beta)}{[1-\gamma r(\alpha+\beta)]} \left(\frac{P^N\ \bar{\theta}\ \alpha^{\alpha}\beta^{\beta}}{w^{\beta}\ r\ r_k^{\alpha}} \right)^{\frac{1}{1-\alpha-\beta}} p^{\frac{1}{1-\alpha-\beta}} \tag{1.31}$$

$$
\begin{aligned}
i_{nt}(e_{nt}(p)) &= \frac{r}{p} \\
M_{nt}(e_{nt}(p)) &= r_k k_{nt}^N(p) + w l_{nt}(p) - e(p)
\end{aligned}
$$

Proof. *See Appendix.*

Interestingly, the amount nanced internally under a truth telling contract increases with p, parameter that also represents the size of the project, and with γ, indicating that banks will lend proportionally more when entrepreneurs are more impatient. Note that by letting $\gamma r = 1$, the net worth required becomes

$$
e(p) = (\alpha + \beta) \left(\frac{P^N \ \bar{\theta} \ \alpha^\alpha \beta^\beta}{w^\beta \ r \ r_k^\alpha} \right)^{\frac{1}{1-\alpha-\beta}} p^{\frac{1}{1-\alpha-\beta}} \tag{1.32}
$$

which is the total cost of production for a rm with characteristic p.[21] This implies that $M_{nt}(e_{nt}(p)) = 0$: the owner will only have incentives to reveal his characteristic when there is no borrowing! When the subjective discount rate is higher than the interest rate, the bank will be able to make a truth telling loan contract since only those rms with a high enough probability of surviving are willing to postpone consumption to invest in the rm.

Again, these contracts are only truth telling if the rm take the same contract in

[21] This can be seen by computing the total cost as

$$
TC_{nt}(p) = \left(r_k k_{nt}^N(p) + w l_{nt}^N(p) \right)
$$

the future, which happens in equilibrium. Otherwise, the asymmetry of information would persist because there would be incentives for the lower types to mimic good types knowing that they would get better contracts in the future (contracts that allow them to invest less and get the same lending rate).

Entrepreneurs will qualify for this last type of contract only if they have enough wealth. Since all entrepreneurs in each cohort start with the same net worth, high quality types the ones with more productive potential spend more periods without being able to engage in truth telling contracts. What do they do then?

Without the appropriate level of wealth, rms end up engaging in nancial contracts that are not truth telling. Their problem is to maximize (29) subject to Equation (30). As it was mentioned before, all those $p > p^*_{nt}$ will participate, and the problem reduces just to pin down p^*_{nt}.

Every rm that has not taken a truth telling contract in the past, chooses between participating in a pooling contract and participating in a truth telling contract (contingent on having enough net worth). Note that in principle the bank can set up different pooling contracts (for different best quality types in different pools). Nonetheless, the following statement holds.

Proposition 5 *In equilibrium, every entrepreneur that belongs to the some cohort with characteristic $p \geq p^*_{nt}$ and with the same net worth will participate in the same pooling contract.*

Proof. *See Appendix.*

Corollary 1 *All entrepreneurs in the some cohort that belong to a pooling contract will have the same net worth.*

Proof. *See Appendix.*

This Corollary follows from Proposition 8. Given that everybody participates in the same pooling contract as long as they don t take a truth telling one, and that all entrepreneurs in the same pool started with the same net worth (coming from labor endowment), we get the result that everybody that succeeded in the past will have the same net worth independently of their type.

Thus entrepreneurs with quality $p > p^*_{nt}$ will take a pooling contract if and only if total return under the pooling contract is at least equal to total return under the truth telling one. The lowest type can choose to take the latter type of contract only when her net worth is big enough. Thus,

$$TR_{nt}(e_{nt}, \bar{p_{nt}}\,(p^*_{nt}), p^*_{nt}) \geq TR_{nt}(e(p^*_{nt})) + \frac{1}{\gamma}[e_{nt} - e(p^*_{nt})] \tag{1.33}$$

The return under a pooling contract depends on the average quality $\bar{p}_{nt}$, which is obviously a function of the worse type p^*_{nt} and is based on total wealth of the best entrepreneurs in the cohort (since they are willing to invest as much as they have, $NW_{nt} = e_{nt}$). Total return under truth telling is the sum of the return from the rm, based on net worth $e(p^*_{nt})$, and the return coming from utility (or consumption), based on $NW_{nt} - e(p^*_{nt})$, which is consumed right away.[22] It is worth highlighting that the participation constraint in Equation (33) only takes into account the present trade off between free riding and taking a truth telling contract. This occurs because under reasonable assumptions regarding the density function $f(p)$, an entrepreneur that is indifferent between free riding or taking a separating contracts (one with characteristic p^*_{nt}) will strictly prefer to reveal himself tomorrow, since the wealth of the best entrepreneurs in the cohort that survive one more period will be even greater, and they will be willing to re-invest all their revenues[23]. This implies that if there are no gains from free riding on today s pooling nancial contract, there won t be any gains from free riding on tomorrow s pooling contract. A simple proof of consistency to see whether Equation (33) is the right participation constraint is to check $p^*_{nt} < p^*_{n+1t+1}$.

By using $e(p^*_{nt})$ from Equation (31), plugging it into the last expression and sim-

[22] It can be trivially proved that no entrepreneur has incentives to undertake a truth telling contract for a type worse than her own. By staying in the pool she will get a subsidy until it is optimal for her to truthfully reveal her own characteristic. And all these contract are cheaper than the one she could get by mimicing a lower type.

[23] If under the present speci cation we get that for some t, $p^*_{nt} > p^*_{n+1t+1}$ it means that at time t there was a type below p^*_{nt} that would have prefered to choose to free ride on the pooling contract. These cases, although they can be handled, only happens under extreme assumptions on $f(p)$ since it has to be the case that the average quality of rms in the pooling contracts sharply increases between t and $t+1$ even for the same p^* (see Proposition 4). Thus, we need a lot of mass on low values of p since $f_{n+1t+1}(p) = pf_{nt}(p)$.

plifying we are able to get the participation constraint.

$$\left(\frac{P^N\ \bar{\theta}\ \alpha^{\alpha}\beta^{\beta}}{w^{\beta}\ r\ r_k^{\alpha}}\right)^{\frac{1}{1-\alpha-\beta}}\left[\bar{p}_{nt}^{\frac{\alpha+\beta}{1-\alpha-\beta}}-\frac{(1-\alpha-\beta)}{(1-r\gamma(\alpha+\beta))}p_{nt}^{*\frac{\alpha+\beta}{1-\alpha-\beta}}\right]\geq e_{nt}\frac{(\bar{p}_{nt}-r\gamma p_{nt}^{*})}{(1-\alpha-\beta)r\gamma p_{nt}^{*}\ \bar{p}_{nt}} \tag{1.34}$$

The participation constraint will be always binding in the steady state. Those types participating in the pool today are only the ones that were in the pool in previous periods (unless this is a newborn cohort). While in the steady state this constraint always holds with equality (regardless of the age of the cohort), when the economy is out of the steady state after a shock for example the entrepreneur s net worth can be so low that every member of the cohort that was in the pool in the previous period will be willing to participate in it today. I ll come back to this point later.

Also it is worth noting that if e_{nt} becomes high enough then this equation will only hold for $p_{nt}^{*}=\bar{p}_{nt}=1$. The net worth level that makes the participation constraint binding for a lowest type $p_{nt}^{*}=1$ is given by:

$$\left(\frac{P^N\ \bar{\theta}\ \alpha^{\alpha}\beta^{\beta}}{w^{\beta}\ r\ r_k^{\alpha}}\right)^{\frac{1}{1-\alpha-\beta}}\left[\frac{(1-\alpha-\beta)r\gamma(\alpha+\beta)}{(1-r\gamma(\alpha+\beta))}\right]=e_{nt} \tag{1.35}$$

which is the net worth required by a truth telling contract to an entrepreneur announcing $\hat{p}=1$!

More generally, the following result holds.

Proposition 6 *The lowest and average type participating in a pooling nancial contract, p^*_{nt} and $\bar{p}_{nt}$ are non-decreasing functions of the entrepreneurs net worth e_{nt}.*

Proof. See appendix.

Proposition 10 means that as the amount nanced internally increases, the average quality of the pool improves. This happens because incentive problems between low quality rms and banks decreases when rms put more at stake in the investment project.

In the next subsection I describe how the interest rate shocks impact on the price of non-tradeable goods, and hence on the rms revenues.

1.4.2 Macroeconomic effects of the shock

As it was mentioned before, technology in the non-tradeable sector is given by a constant returns to scale production function. Moreover, assumptions on technology in this sector allow us to state rst order conditions as follows.

$$AF_{Y^N} = r_t P_t^N \tag{1.36}$$

$$AF_k = (r_t - 1 + \delta^T) \tag{1.37}$$

Let r_l and r_h be the interest rates in normal and crisis times respectively, the next result follows.

Proposition 7 *If labor supply is infinitely elastic, there is only one possible equilibrium non-tradeable good price corresponding to each in interest rate, $P^N(r_l)$ and $P^N(r_h)$ with $P^N(r_l) \geq P^N(r_h)$.*

Proof. See Appendix A.

The fall in the intermediate non-tradeable output price has two effects. On one hand, it surprises firms that were expecting good macroeconomic conditions and high prices. The fall in the non-tradeable price triggers a net worth effect in the non-tradeable or bank dependent sector. On the other hand, it increases the exit probability for small firms. Both together put the economy in a recession because it takes time for surviving firms to recover their net worth and for the economy to replace the firms that exit with good financial reputation. The severity and duration of this effect depends on parameter values.

1.4.3 Financial contracts out of the steady state

It is worth noting that Propositions 4 to 10 also hold out of the steady state. In particular, even though for some type of entrepreneur her future net worth might not be big enough to satisfy the financial contract given in Proposition 7, after a shock for example, banks will finance the firm as long as the entrepreneur invests all her wealth. This situation continues until net worth is reestablished to normal levels.

Would this be violating the commitment undertaken by banks in previous period? The answer is no. The purpose of the commitment is to avoid having some types be free ridden by worse ones. After the shock, the banks can renegotiate the truth telling contracts because the expected probability of such shocks is negligible, implying that no agent was expecting it. Thus, even when the banks renegotiate with rms after a shock, that fact that this shocks are unexpected make Equation (33) the correct participation constraint before the shock.

Note that if the entrepreneurs net worth collapses to zero, everybody will want to participate since Equation (34) hold with strict inequality because the left hand side of this expression is always positive,[24] even for a type $p^*_{nt} = 0$. Nonetheless even if the net worth of all entrepreneurs in the same cohort collapses to zero, not all members of the same cohort will be taking the same pooling contract because banks learn the productivity of their clients by observing the type of contracts they took in the previous period. Then, banks can distinguish those that took a pooling contract in the previous period and will offer them a nancial contract using this information. This implies that once the bank knows that a certain type has characteristic p bigger than p^*_{n-1t-1} , they will never offer them a contract where a type lower than p^*_{n-1t-1} is willing or able to take.

1.4.4 Equilibrium

[24]See that $\frac{(1-\alpha-\beta)}{(1-r\gamma(\alpha+\beta))} \leq 1$ by assumption and that $\bar{p}_{nt} \geq p^*_{nt}$ for all cohorts.

I have shown that the nancial contracts proposed are equilibrium contracts both in and out of the steady state. Now existence of equilibrium follows by showing that the allocations derived from this contracts describe well behaved aggregate excess demand functions for all goods in this economy.

Proposition 8 *Equilibrium exists for an economy* $(\mu, \{(\eta_{nt})_{n=1}^{\infty}\}_{t=1}^{\infty}, f(p))$ *both in and out of the steady state.*

Proof. *See Appendix A.*

1.5 Simulations

In this section I rst set up the parameters of this model to then carry a comparison between three simulation exercises. The rst exercise has the property of switching the reputation mechanism off so we can focus on the implications of extending the Bernanke and Gertler s net worth approach to a dynamic setting where rms live for many periods. This is done by letting all rms have the same survival probability after the shock on interest rates as in normal times.

The second simulation exercise differs from the benchmark case in that the survival probability changes on impact as shown in Section 2. The third simulation exercise is similar to the second, but it also includes an externality in the tradable sector.

Lastly, the model allows me to analyze the microeconomic performance of all types of entrepreneurs not only in the steady state, but also after a shock. This information is a by- product of the model, which requires computing for nancial contracts at each period.

1.5.1 Parameters

The tradable goods production function adopted for the simulation is a standard CES

$$Y_{t+1}^{T} = A\left[\phi\left(K_t^T\right)^{-\rho} + (1-\phi)\left(Y_t^N\right)^{-\rho}\right]^{-\frac{1}{\rho}} \tag{1.38}$$

where $A > 0$, $\rho > -1$ and $\phi \,\epsilon\, (0,1)$, where $\frac{1}{1+\rho}$ is the elasticity of substitution between capital and non-tradeable inputs.

The parameter values were chosen to roughly match shares of labor and capital in total output for Argentina and to produce a fall in the intermediate good s prices of 10% as a response to a strong shock on the interest rate. These parameters are listed in the following table

$A \cong 1.349$

$\rho_1 = 7.0678$

$\phi = 0.2039$

For simulation purposes the interest rate levels are $\{r_l, r_h\} = \{1.0147, 1.035\}$.

The elasticity between capital and the non-tradeable good is required to be low enough to generate a fall in prices of approximate 10% and a fall in the capital stock of only, say 4%. This high complementarity can be relax at the cost of increasing the volatility of investment in the tradeable sector.

The sum of the distributional parameters on the non-tradeable production function was set as large as possible given Assumption 1. This matches microeconomic evidence for the US about technology at the plant level, rms growth and evolution of nancial sources.[25][26] Thus, $\alpha = .35$ and $\beta = .61$, capturing the idea that small rms are labor intensive. The Solow parameter in this sector ($\bar{\theta} = 3.12$) was chosen to obtain the result that the labor demanded by the biggest rm be 150 times the labor demanded by the smallest rm in the non-tradeable sector, where this ratio was arbitrarily chosen.

Parameters for the worker s utility function are given in the following table.

$a_1 \cong 2.06$

$b_1 = .33$

$\sigma = 3$

The parameters corresponding to the labor supplied in the non-tradeable sector were calibrated to normalize steady state wages in this sector to one and to match evidence that labor elasticity is equal to $\frac{1}{2}$ in developing economies.[27] Finally, the intertemporal elasticity of substitution, represented by the parameter σ is assumed to have a value of three, to mimic evidence in emerging economies.

The mass of workers μ is set in the following way. For xed wages and non-

[25] Since I wasn t able to obtain microdata from developing economies, I took evidence for the US as a gross substitute to it. Future research should address this question.

[26] See Cooley and Quadrini (1998) and Davis, Haltinwanger and Schuh (1996) for a discussion on these issues.

[27] See Rebelo and Vegh (1995).

tradeable prices, total labor demand in this sector is given. To normalize labor supplied by each worker to one, I let the mass of workers be equal to labor demand minus labor supplied by entrepreneurs. For simulation purposes I assume that $\Gamma_0 = 0$, no initial wealth is held by workers, meaning that all workers wealth comes from wages.

The entrepreneur discount rate was chosen to match a reasonable leverage level for a rm that has solve all agency problems (the biggest rm for example) and letting it be bigger than the interest rate at all times, good or bad. Thus, $\gamma = \frac{1}{r_h + .01}$.

The density function utilized in this numeric example is $f^1(p) = 6p(1-p)$, where the numbers were set to let the function integrate to one and to match reasonable average spreads between deposit and lending rates. From this density function, it can be seen that there is no mass of rms with characteristic parameter one or zero, implying that everybody produces something and that no rm leaves for ever.

Finally, I assume that the probability that rms exit the industry for non- nancial reasons is 2% in steady state and 4% on impact. I believe these are conservative rates for a small emerging economy like Argentina.

Finally, depreciation rates for capital in the tradable and non-tradeable sectors were arbitrarily xed at 6%. Results in the model have shown to be robust to different depreciation rates, although lower depreciation rates require higher complementarity between capital and non-tradeable inputs in the tradable production function to be able to produce a 10% drop in non-tradeable prices on impact.

1.5.2 Three simulation exercises

Before entering into the actual comparison of the three simulations, I present the nature of the externality assumed in this section. This externality is introduced by letting the total factor productivity in the tradable sector depend on aggregate non-tradeable output. For concreteness, I assume

$$A_t(Y_{t-1}^N, \bar{Y^N}) = A\left[1 - \nu\left(\frac{\left|\bar{Y^N} - Y_{t-1}^N\right|}{\bar{Y^N}}\right)\right] \tag{1.39}$$

where $\nu > 0$. For any scale bigger or lower than the long run aggregate non-tradeable output scale, $\bar{Y^N}$, tomorrow s total productivity decreases. The idea behind this assumption, is that the non-tradeable output is a composite of many different goods that are needed for production. When the economy enters in a recession, and the amount produced decreases, the marginal productivity of tomorrows tradable sector decreases due to coordination problems between sectors, adjustment costs, etc.

The parameter ν determines the relative importance of the externality. Because obtaining a measure for this parameter is difficult I approach the problem in the following way: I pick a parameter value that do well in matching the evolution of aggregate output in this small open economy. In this simulation I have adopted a parameter $\nu = .25$, implying that a one percent drop in total non-tradeable output at t decreases total factor productivity by 0.25% in the period that follows.

I compare the macroeconomic performance of these three models in one dimension:

aggregate tradable output. It is worth noting that across the three cases, all variables are the same in the steady state, since there is no bankruptcy of rms with high output performance and there are no externalities because total non-tradeable output is being produced at its long run scale.

The comparison can be observed in the following chart, where Model 1 refers to case where only the net worth channel is at work, Model 2 refers to the model where the exit rate increases, and nally Model 3 is equivalent to the second case adding externalities to the economy. Also, just as a theoretical exercise, I show the evolution of total output when only the externalities are present (Ext.). This is done by letting all rms have the same net worth on impact instead than in the steady state and the same survival rates.

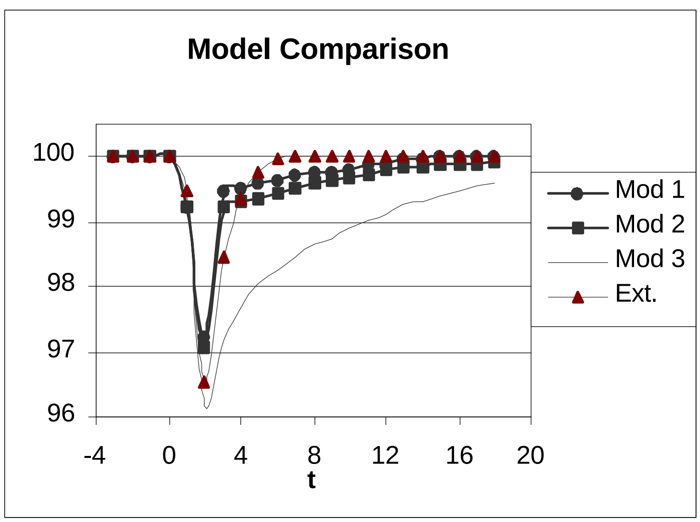
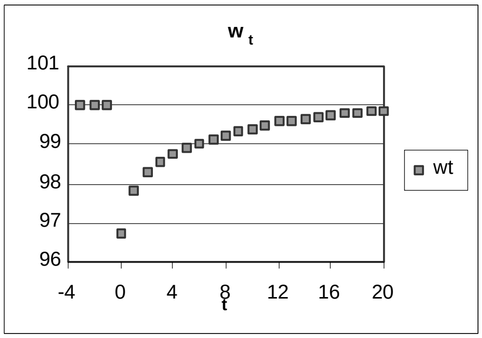
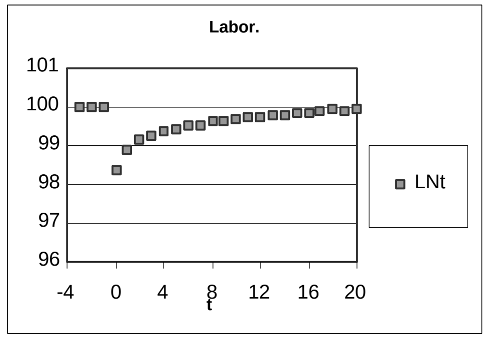
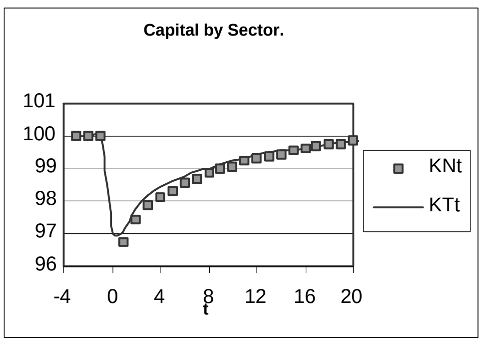


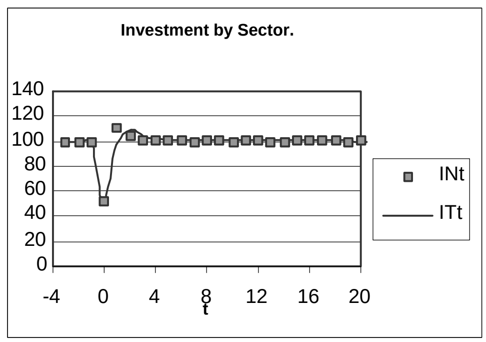
Figure 1.5: Model comparison

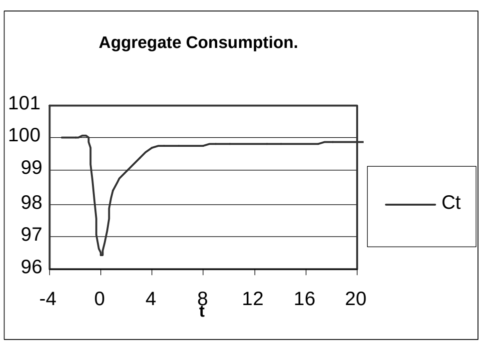
The model with externalities and bankruptcy is capable of producing more severe business cycles downturns for the same interest rate shock even though externalities alone have very weak serial correlation. Because models without externalities underestimate the business cycles experienced by these economies, I continue by presenting all the macroeconomic variable simulated under this last case. All the main macroeconomic variables are presented in the graphs below.

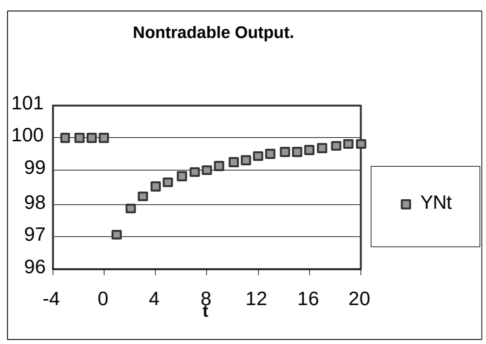












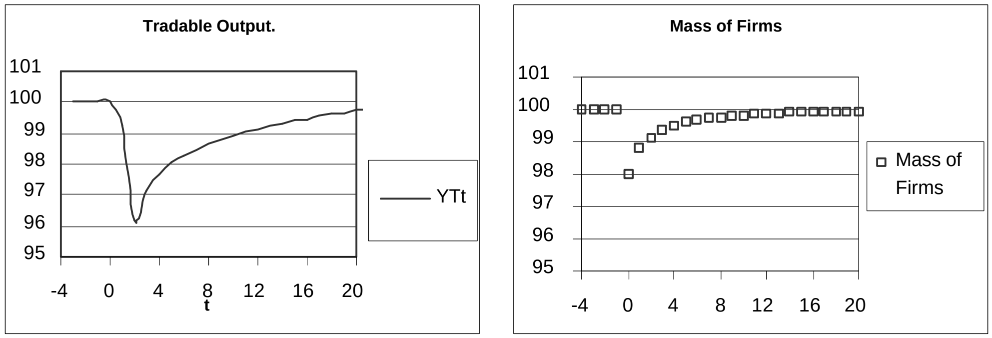


Figure 1.6: Simulated macroeconomic aggregates

The simulation, as in the previous cases, was done by assuming that the interest rate increases at period 0, and it returns to normal levels right away. Wages in the non-tradeable sector are pro-cyclical. Employment decreases as a response to lower wages. Capital in both sector decreases on impact due higher interest rate. After the shock, capital remains low because non-tradeable output is lower than under steady state, and the two are highly complementary by assumption. Investment in both sector drops sharply on impact and then increases so that capital steadily recovers its steady state level. Aggregate consumption is mostly workers consumption, and it is highly correlated with output.[28] This is due the assumption on workers preferences since the sum of both leisure and consumption are smoothed out over time. Non-tradeable output decreases on impact and it remains depressed through many periods. This is due to externalities: exiting and agency problems between banks and rms. I come back to this last point below. Tradable output is temporary reduced after the

[28] Although it also includes entrepreneurs consumption, this is around 2.5 percent of aggregate consumption.

shock since it takes time for the economy to recovery due to problems in the non-tradeable sector. Finally, the mass of rms drops 2% by assumption and although it recovers quickly, tradable output doesn t recover because it takes time for new good rms to build up their net worth and thus to get lower interest rates in nancial contracts.[29]

The simulations show that shock propagates through time despite the fact that this shock happens only at $t = 0$. Wages, tradable and non-tradeable output, investment and consumption experience depression an it takes a while for the economy to return to its full potential output and consumption levels. This model shows how externalities, exiting and agency costs drive the cycle after the shock.[30]

Higher agency costs are incurred through two informational channels. The rst channel which I call net worth mechanism takes place when all rms experience losses after a bad shock; the result is that wealth is drastically reduced, and that the proportion of free riders within the same pool becomes higher than it would otherwise be. The main reason for this is that incentive problems between rms and banks are positively correlated with leverage, which is much bigger after the bad shock since rms are nancially devastated.

The second channel which I call the reputational mechanism is due the loss of information when exit occurs. The rms that exit due to the macroeconomic shock

[29] The trade balance is sharply improved on impact mostly due to the drop in investment.

[30] The interest rate paid by rms increases during the recession in this model economy.

destroy not only present but also future output since the production levels of exiting firms can only be regained once younger generations pass through the costly screening process of producing over time. Again, this process is costly because younger firms with a high productivity parameter are unable to convince banks to finance large investment projects since firms similar in age and equity but with a low productivity parameter have private incentives to free ride on those contracts.

Due to these agency problems in the non-tradeable sector, the shock puts the economy in a long-lasting and recessional path, a situation that is aggravated by the presence of externalities. While this externalities were chosen to contribute to the economic downturn by only 25%, equilibrium effects are stronger, because pecuniary externalities are also important in the model. When total factor productivity decreases, non-tradeable prices are also reduced, driving non-tradeable output down with it. When externalities are present it takes even longer for firms to recover their net worth. This is the reason why externalities add so much to the business cycles.

To complete the analysis of the model, in the next Subsection I present some microeconomic information drawn from the simulation.

1.5.3 Microeconomic information

In this subsection I present rm data simulated for the model with externalities, both in and out of the steady state.[31] To analyze this information in the steady state, it is better to concentrate on the data generated by a rm owned by an entrepreneur with the highest characteristic parameter p. Remember that this entrepreneur keeps his productivity over time, as long as he is productively successful. The graph below shows the main rm variables as a function of the age of the rm (per quarter), assuming the economy is at its steady state (or prices of inputs and output are constant).

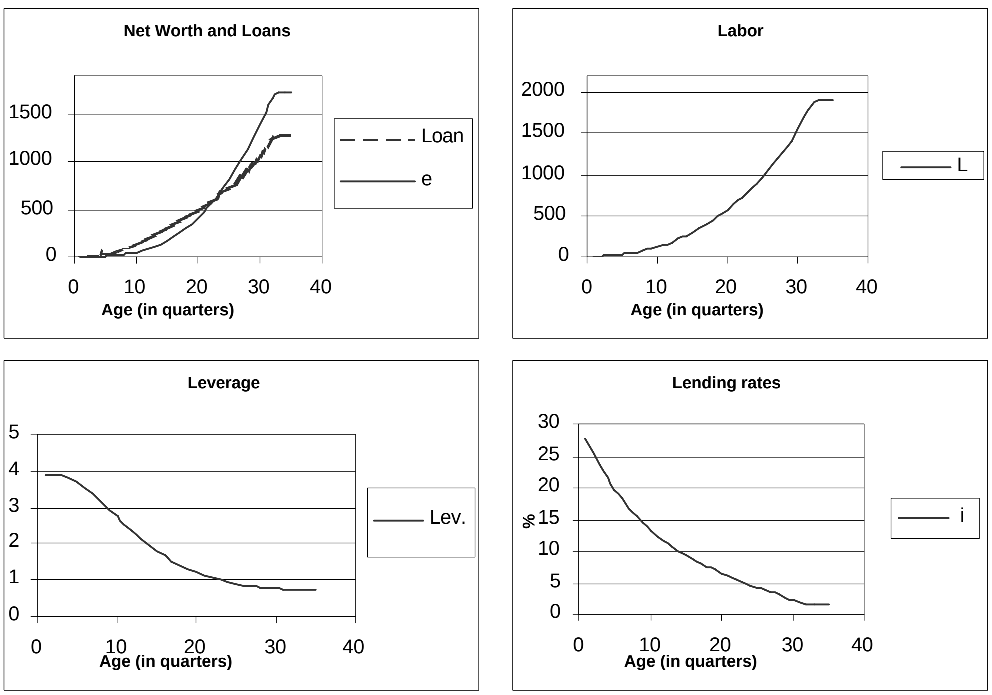


Figure 1.7: Simulated microeconomic variables in steady state

Net worth and amounts loaned are positively correlated, evidence that the banks

[31] Again, microeconomic data corresponding to steady state levels are the same for the three models.

utilize the rms wealth as a revealing informational screening device. It is worth mentioning that in a symmetric informational environment, these variables would not be correlated. Also they increase with age, since by assumption the (highest quality) rm is productively successful in all these periods. The net worth has an upper bound because of the assumption that technology in this sector exhibits decreasing returns to scale. The simulation shows that only after 33 quarters, these rms are able to take truth telling contracts that fully solve the asymmetric information problem with banks.[32]

Inputs and output also increase with age, as can be observed in the graph for labor demanded by rms.

Leverage, expressed as the ratio of loans to net worth, is monotonically decreasing with the rm s age. As rms get older, the fraction of spending that is self- nance converges to the fraction in truth telling contracts, meaning that net worth grows proportionally faster than bank loans in the rm s rst periods of life. This fraction stabilizes once the rms take truth telling contracts.

Finally, the interest rates paid on loans by these rms decreases with age as the bank s perception of the rms quality improves. Younger rms pay higher rates because their reputation -and their access to credit markets- has not been developed.

In the next graphs I present micro-data for the simulation with externalities after

[32] Note that the performance of a low-type entrepreneur gives a similar graph except that the pooling nancial contracts would be dropped at an earlier stage. This statement holds by Corollary 8.

the economy was hit by the external shock. When the economy is at its steady state, as in the previous graphs, time series micro-data coincides with cross sectional data. In contrast, after the economy is impacted with a high interest rate, time series and cross sectional data differ because a rm s performance will depend on the age of the rm at the moment of the shock. In the next set of graphs, I show time series data for the highest quality rms that belong to a ve period old cohort at impact. For comparison, I present the information on this cohort as a ratio of actual data to the time series data that would have been produced by these rms if no shock had occurred.

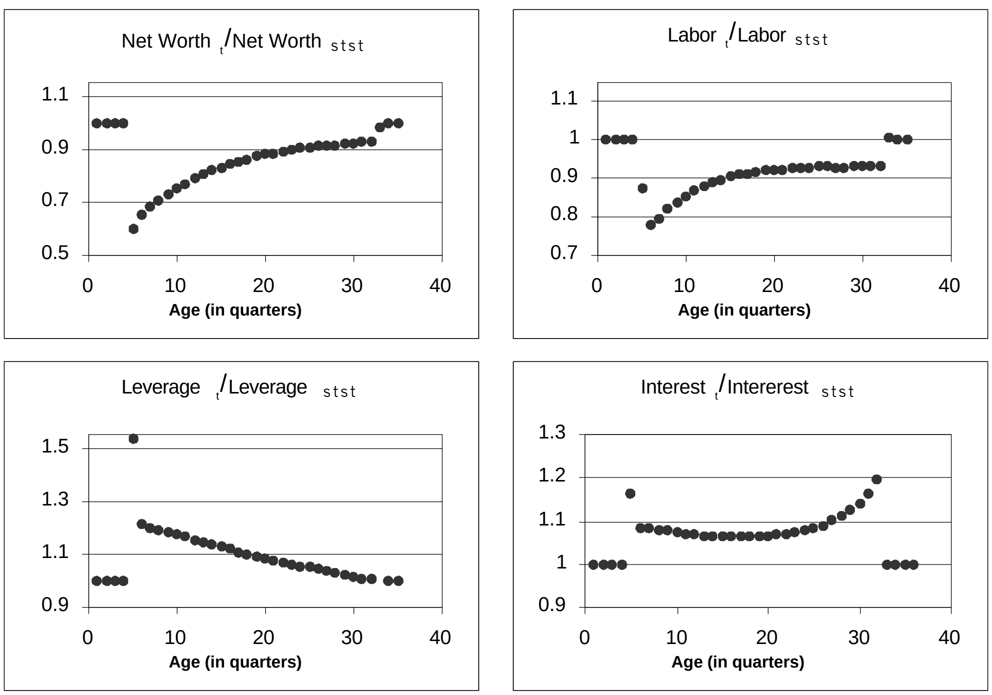


Figure 1.8: Simulated microeconomic variables along the business cycle

The ratio of actual net worth to that in steady state conditions is lower than one, showing that rms that are hit by the shock will only recover after 35 periods.

During this time, labor demand will also be lower since agency costs are higher. Actual leverage is temporary higher than the steady state level of leverage in all these periods, because banks do no require rms to nance in the same proportions as at steady state, since some information about this cohort average quality has been already revealed. Clearly, the information revelation process takes longer in recessions due to the net worth effect. Finally, the ratio of actual interest rates paid by rms during recessions to those rate paid in the steady state are higher throughout the recession.

1.6 Policy analysis

The size of the economic recession, given assumptions on technology and preferences, is in direct relationship with the size of the external interest rate rise. Higher rates imply lower unexpected non-tradeable prices in the economy and this will increase the exit probability and reduce even further the net worth of those rms still producing. The deeper the interest rate crisis the deeper and longer lasting the recessions due to higher agency costs, exiting rates and externalities, leaving room for policy analysis.

The shock reduces welfare in two different ways. On the one hand, workers have a cost in terms of expected welfare because their utility function is concave in the sum of consumption and leisure, and the shock reduces expected utility by Jensen s inequality. On the other hand, agency problems add welfare costs to both entrepreneurs and workers since pro ts and wages are reduced throughout the economic downturn.

Any stabilizing policy that neutralizes sudden changes in entrepreneurs wealth might improve the overall performance of this economy. Thus there are different policies that might be implemented. A subsidy to the interest rate in bad states or any policy that in ates the demand in the non-tradeable sector will help to reduce the recession. A sterilization policy, used directly or indirectly in emerging economies, implies that the interest rate is subsidized when the bad shock occurs. Then, the government should collect taxes in good times and subsidize interest rates in bad times. This can be done even if taxes are collected after the subsidy takes place.

Under such a policy total welfare would be greatly increased. A rst order measure of welfare gains can be approximated as the area delimited by the full capacity level and the actual performance of the economy s tradable output along the cycle in net present terms.[33]

A more realistic policy would be one where the government collects liquid international resources in good times to subsidize interest rates in bad times[34]. This policy can be implemented at the cost of keeping productive resources underutilized. The cost of keeping these reserves will determine the optimal degree of intervention in

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[33] Note that this measure is a lower bound on the total welfare gains of this stabilization policy since this policy also increases expected worker s utility given the concavity assumption on their preferences.

[34] The IMF has pact contingent loans with small emerging economies that have the same purpose than the policies proposed here.

each economy.

Regardless of the intervention levels, such a policy might always be welfare improving in economies (or episodes) that face severe and unexpected increases in interest rates without including the cost of the policy since the aggregate agency cost is a monotonic function of the change of these rates.

1.7 Concluding remarks

This paper shows that nancial frictions may be a strong transmission mechanism for the propagation of shocks in small open economies, both from the qualitative and quantitative points of view. Of all the different ways to model nancial frictions developed in the literature, I took that proposed by Bernanke and Gertler (1990) since the asymmetric information problem they emphasized seems the most appropriate and representative one in nancial relationships. When this friction is incorporated into a dynamic macroeconomic model, we obtain two effects that impact the incentive side of nancial contracts and hence impact the macroeconomic performance of the economy: net worth and reputation . The rst one was analyzed by Bernanke and Gertler (1990) in a static environment, concluding that the rms nancial health might have an important role explaining aggregate agency costs and output performance. In this work, I show that although the net worth effect is present and important in a dynamic setting, the reputation effect might be also important when there is information to be learned about rms quality from their performance over time: if rms with good reputations die in the presence of unexpected bad news it takes a long time to replace them.

Under the present setup, most of the macroeconomic variables in the simulation are well behaved. Interest rates and investment in the tradable sector are the leading indicators of the cycle. Low investment levels depress the small rms output price putting rms in a fragile nancial situation since their revenues are lower than expected. This reduces aggregate performance because rms are less able to convince the banks to nance large investment projects. If the shock implies a greater exit probability, then the economy will perform even more poorly for some periods following the shock because rms that have developed a good nancial reputation disappear and it takes time before new rms develop their own. Aggregate performance declines even further in the presence of externalities. This dynamic leads to counter-cyclical agency costs and pro-cyclical employment and consumption. These features of the model match empirical evidence.

In this environment, sterilization policies may be welfare improving depending on the cost of implementation. Neutralizing capital volatility when it can be done at a relatively low social cost will help the economy to perform more closely to its full productive potential. From the theoretical point of view, any policy that reduces the non-tradeable price uncertainty would improve total welfare.

1.8 Appendices

1.8.1 Appendix A

The Industrial Production Index (with 1986=100) includes the following industries: Food, Beverages, and Tobacco, Aparel, Paper, Chemical, Construction, Metallic and Machines and Equipment.

The series presented in the Introduction is the Industrial Production Index for Argentina and modi ed as follows. First, I replaced all February s observations by the average of January s and March s observations, since the Index exhibits a sharp decline on each February due to vacations.[35] Second, I computed a linear trend for two periods: February 1992 to December 1994 and March 1994 to December 1994. I utilized the second linear trend since it is the most conservative one (not show in the Graph below). The Graph shows an exponential and a linear trend based on the period 1992-1994, as well as a linear trend based on the period 1992-1998. The graph shows that by computing deviations from the linear trend based on the period 1992-1994, the Industrial Production Index would not recover trend until September 1997. This implies an even longer recession than the one presented in the Introduction.

[35] Leaving the Index intact would increase the trend rate (ctitiuosly), reinforcing the argument that the economy entered in a long recession.

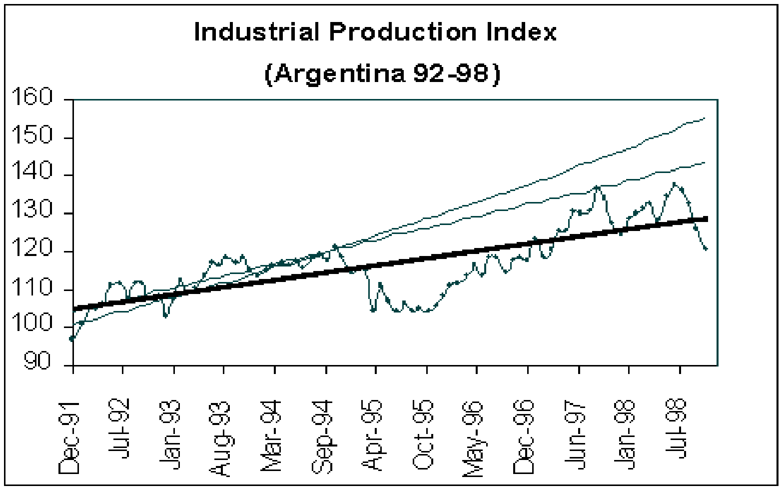

Figure 1.9: Alternative detrending hypothesis

The series for the average deposit interest rates of commercial banks in the Argentinean nancial system are build as a weighted average of the average interest rate paid to deposits denominated in pesos and in dollars.

The average lending rate is the weighted average interest rate charged to loans denominated in pesos and in dollars to local rms, big and small. There is no information on the interest rates paid on bank loans by small rms neither on loans to small or AAA rms. This is a problem since in this article I focus on the dynamics of small rms along the business cycles.

As a reference, I present the spread between the average deposit rate and the average lending rate of the nancial system.

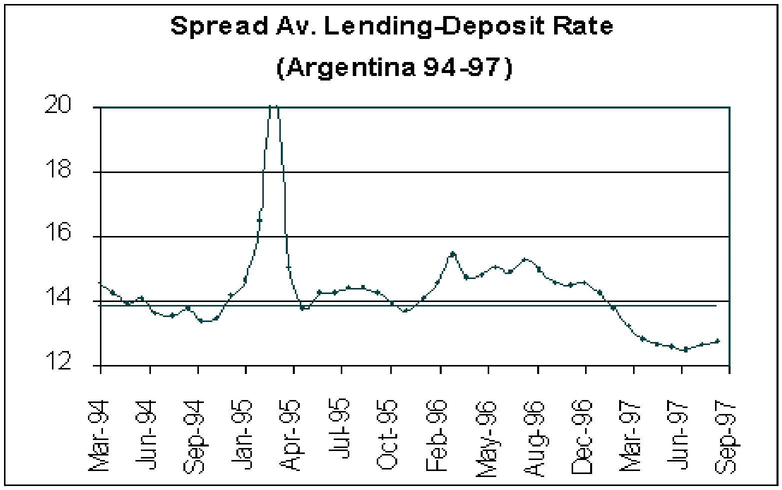

Figure 1.10: Lending premium

As the graph shows, the spread returned to normal levels right away after the sharp spike on impact. It seem that the persistence in the lending premium is nonexistent. Nonetheless, this result is driven by changes in the composition of lending to small and AAA firms during the downturn.

By definition the average lending rate, regardless of denomination issues, is

$$r_t^L \equiv \alpha_t r_t^S + (1 - \alpha_t) r_t^b$$

where the interest rates are the average lending rate, the lending rates for loans to small firm and big firms (or AAA firms) respectively, and α_t is the fraction of lending to small firms. Rearranging this expression we get the spread in the previous graph.

$$r_t^L - r_t^b \equiv \alpha_t (r_t^S - r_t^b) \tag{1.40}$$

I present this information in the following chart.

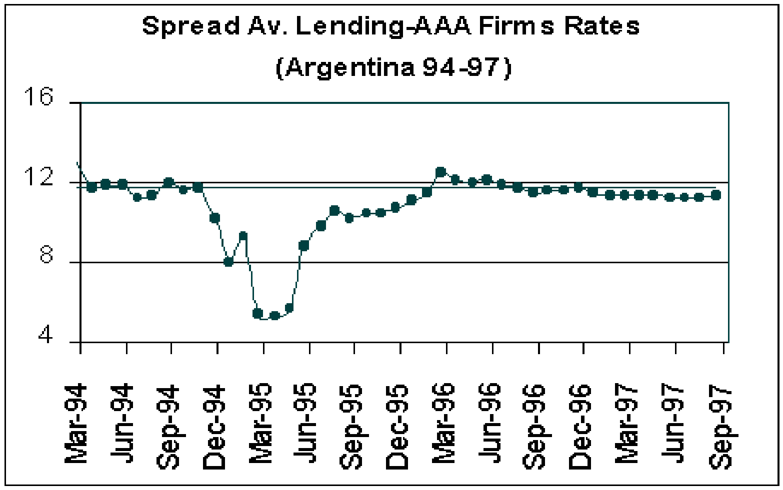

Figure 1.11: Lending rate differential between big and small-medium sized rms

This spread shows that the average lending rate and the lending rate to AAA rms get signi cantly close in the rst months after the interest rate shocks. As Equation (40) shows, this result is consistent with changes in α_t or in $(r_t^S - r_t^b)$ or in both. Although we have an identi cation problem, anecdotal evidence points that small rms face relatively higher interest rates. This implies that the drop in the spread shown in the previous graph must be driven by a sharp fall in α_t. Since overall lending of the nancial system fall over this period, credit to small rms must have fallen even further.

To see that note that in June 1995, the spread between the Average lending rate and the rate for loans to AAA rms is almost 4% below the same spread before and after the crisis. Also, the AAA rate was 5 percentage points above steady state values. If α_t did not fall, then this observations imply that while AAA rate was 5 percentage points above trend, the lending rate for small rms was only 1% above trend. This scenario is refuted by anecdotal evidence.

1.8.2 Appendix B

Proof. of Proposition 4. Dropping subscripts and taking partial derivatives to expression (17) give us

$$\frac{\partial \, \bar{p}}{\partial p^*} = \frac{f(p^*)p^{*n}}{\int_{p^*}^{1} f(x)x^{n-1}dx} \left(\frac{\bar{p}}{p^*} - 1 \right) > 0 \ \forall \ p^* \in [0,1)$$

where it is easy to further show that this derivative goes to one from below as $p^* \rightarrow \bar{p}$. Also,

$$\frac{\partial \, \bar{p}^{Pool}}{\partial n} = \frac{n}{(n-1)} \frac{\int_{p^*_{nt}}^{1} \ f(p) \ p^{n-1}dp}{\int_{p^*_{nt}}^{1} \ f(p) \ p^{n-2}dp} > 0 \ \forall p^* \in [0,1)$$

$\blacksquare$.

Proof. of Proposition 5. Under full information, the rms problem becomes[36]

$$\max_{\{k^N, l^N, i, M\}} E_t[TR(e,p)] = p[P^N \ \bar{\theta} \ (k^N)^{\alpha}(l^N)^{\beta} - iM] \tag{1.41}$$

subject to

$$p \ iM - rM \geq 0 \tag{1.42}$$

$$r_k k^N + wl^N = e + M \tag{1.43}$$

[36] I loose unnecessary notation.

where it can be seen that there is no adverse selection since the bank lends at a rate that take into account the true entrepreneur characteristic p. The solution to this problem is just given by

$$k^N = \left[\frac{pP^N \, \bar{\theta} \, \alpha^{1-\beta}\beta^{\beta}}{w^{\beta} \; r \; r_k^{1-\beta}}\right]^{\frac{1}{1-\alpha-\beta}} \tag{1.44}$$

$$l^N = \left[\frac{pP^N \, \bar{\theta} \, \alpha^{\alpha}\beta^{1-\alpha}}{w^{1-\alpha} \; r \; r_k^{\alpha}}\right]^{\frac{1}{1-\alpha-\beta}} \tag{1.45}$$

where productions plans only depend on each entrepreneurs characteristic and not on the initial net worth e ■.

Proof. of Proposition 6.Let the bank participation constrained and the budget constraint collapse into one equation to solve for the total amount due next period, $i_{nt}M_{nt}$.

$$i_{ny}M_{nt} = \frac{r}{\bar{p}}(r_k k_{nt} + w l_{nt}^N - e_{nt}) \tag{1.46}$$

Plugging this expression into the objective function, simplify the problem to

$$\max_{\{k_{nt}^N, l_{nt}^N, \bar{p}, p^*\}} E_t[TR_{nt}(e_{nt}, p)] = p[P^N \, \bar{\theta} \, (k_{nt}^N)^{\alpha}(l_{nt}^N)^{\beta} - \frac{r}{\bar{p}}(r_k k_{nt}^N + w l_{nt}^N - e_{nt})] \tag{1.47}$$

subject to

$$\overline{p} = E_t[p \mid p \in PC(e_{nt}, i_{nt}, M_{nt}), \; f_{nt}(p)] \tag{1.48}$$

and solution follows from solving this problem ■.

Proof. of Proposition 7. Given a truth telling contract offered by the bank, the entrepreneur solves the following problem today and in every subsequent period:

$$\max_{\{\hat{p}\}} E_t[\pi_t^N(p,\hat{p})] = p\left[(1-\alpha-\beta)\left(\frac{\hat{p}^{\alpha+\beta}\ P^N\ \bar{\theta}\ \alpha^\alpha\beta^\beta}{w^\beta\ r^{\alpha+\beta}\ r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}} + \frac{r}{\hat{p}}e(\hat{p})\right] - \frac{1}{\gamma}e(\hat{p})$$

By taking first order conditions, imposing the truth telling incentive condition ($\hat{p}= p$) and rearranging terms we can obtain the following differential equation on $e(\hat{p})$.

$$(\alpha+\beta)\gamma\left(\frac{P^N\ \bar{\theta}\ \alpha^\alpha\beta^\beta}{w^\beta\ r^{\alpha+\beta}\ r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}}\ \hat{p}^{\frac{\alpha+\beta}{1-\alpha-\beta}} = \frac{r\gamma}{\hat{p}}\ e(\hat{p}) + (1-r\gamma)\ e'(\hat{p}) \tag{1.49}$$

Fortunate enough, a closed form solution to this differential equation exist[37]. Finally, by noting that an entrepreneur with characteristic $p = 0$ never invest ($e(0) = 0$), the proof is completed ■.

Proof. of Proposition 8.This can be easily proved by contradiction. Suppose that are two different equilibrium pooling contracts for types in the same cohort and with

---

[37] This differential equation fits into the following general type of linear differential equations

$$w(\hat{p}) = u(\hat{p})e(\hat{p}) + e'(\hat{p})$$

and its closed form solution is given by

$$e(\hat{p}) = \exp(-\int u\ d\,\hat{p})\ \left(A + \int w\exp(\int u\ d\,\hat{p})d\,\hat{p}\right)$$

the same net worth. Then, one of these type will have and average parameter $\bar{p}$ bigger than the other, implying a lower interest rate on loans. Since types cannot be screened but through age and net worth, being they the same in the two pooling contracts, all entrepreneurs would try to participate in the debt contract that charges lower interest rate. ■

Proof. of Proposition 9.All entrepreneurs in the same cohort start with the same net worth given by labor endowment. This means that rms in a new born cohort participate in the same pooling contract and have the same production plan. Those surviving a period ahead, will have the same net worth regardless of their type. The subset of these taking a new pooling contract will, again end up with the same wealth. This process continues until no pooling contract exist for members of the cohort (until the best quality rms have accumulated enough wealth to take truth telling contract) ■.

Proof. of Proposition 10. Making use of Proposition 1 the proof consist on showing that p^*_{nt} is a non-decreasing function of the entrepreneurs net worth e_{nt}. There are two cases. On one hand, if the participation constraint is not binding, local changes in the entrepreneur net worth does not change p^*_{nt}. On the other hand, when the participation constraint is binding, then p^*_{nt} will change with e_{nt}. Dropping subscripts and rearranging the participation constraint, we obtain

$$e = C\,\frac{p^*\ \bar{p}\left[\bar{p}^{\frac{\alpha+\beta}{(1-\alpha-\beta)}} - \frac{(1-\alpha-\beta)}{(1-r\gamma(\alpha+\beta))}p^{*\frac{\alpha+\beta}{(1-\alpha-\beta)}}\right]}{(\bar{p} - r\gamma p^*)}$$

where C is a constant that depends on parameter values. Call [1] the expression between brackets. Differentiating the participation constraint with respect to p^*, and simplifying gives as

$$\begin{aligned}\frac{\partial e}{\partial p^*} &= \frac{\bar{p}}{(\bar{p} - r\gamma p^*)}\left[\bar{p}\,[1] - (\bar{p} - r\gamma p^*)\frac{\alpha+\beta}{(1-r\gamma(\alpha+\beta))}p^{*\frac{\alpha+\beta}{(1-\alpha-\beta)}}\right] + \\ &\quad \frac{\partial\,\bar{p}}{\partial p^*}\frac{p^*}{(\bar{p} - r\gamma p^*)^2}\left[-r\gamma p^*[1] + (\bar{p} - r\gamma p^*)\frac{\alpha+\beta}{(1-\alpha-\beta)}\,\bar{p}^{\frac{\alpha+\beta}{(1-\alpha-\beta)}}\right]\end{aligned}$$

Where $r\gamma \leq 1$ by assumption. Now, let [2] and [3] be the rst and second expressions between brackets in this derivative. The proof follows by showing that this two expression are positive for all possible values of p^*. Since $\frac{\partial \bar{p}}{\partial p^*}$ is always positive, then $\frac{\partial e}{\partial p^*} > 0$ for all values of p^*.

Rearranging terms, [2] becomes

$$\begin{aligned}[2] &= \frac{\bar{p}^{\frac{1}{(1-\alpha-\beta)}}}{(1-r\gamma(\alpha+\beta))}\left[(1-r\gamma(\alpha+\beta)) - x^{\frac{\alpha+\beta}{(1-\alpha-\beta)}} + r\gamma(\alpha+\beta)x^{\frac{1}{(1-\alpha-\beta)}}\right] \; or \\ &\quad \frac{\bar{p}^{\frac{1}{(1-\alpha-\beta)}}}{(1-r\gamma(\alpha+\beta))}F(x)\end{aligned}$$

where $x = \frac{p^*}{\bar{p}} \in [0,1)$. It is easy to show that $F(0) > 0$, $F(1) = 0$, and $F'(x) < 0 \; \forall x$. This implies that $[2] > 0$. Similarly,

$$[3] = \frac{\bar{p}^{\frac{1}{(1-\alpha-\beta)}}}{(1-\alpha-\beta))}\left[\frac{r\gamma(1-\alpha-\beta)^2}{(1-r\gamma(\alpha+\beta))}x^{\frac{1}{(1-\alpha-\beta)}} - r\gamma x + (\alpha+\beta)\right] \; or$$

$$\frac{\bar{p}^{\frac{1}{(1-\alpha-\beta)}}}{(1-\alpha-\beta))}G(x)$$

where x is de ned as before. Now, $G(0) > 0$, $G(1) \geq 0$, and $G'(x) < 0$ $\forall x$. ■

Proof. of Proposition 11. See that the indirect pro t function for this rms is only a function of P_t^N and r_t. Zero pro t condition $\pi_t^T(P_t^N, r_t) = 0$ implies that there is one possible price of non-tradeable goods corresponding to each interest rate level. If $r_l < r_h$ are the interest rates in normal time and crisis time, then $P^N(r_l) \geq P^N(r_h)$.

Proof. of Proposition 12.

In a small open economy there is no need to for excess demand for tradable goods to be zero. Then we only worry about aggregate excess demand for non-tradeable goods (Y^N) and labor. The aggregate demand for non-tradeable goods is well behaved with respect to P_t^N and w_t, and so is the aggregate supply of labor. Thus, we just need to show that aggregate supply of Y^N and aggregate demand of labor are well behaved functions of P_t^N and w_t.

First, note that each rms supply of non-tradeable output and demand of non-tradeable skilled labor are not continues functions of prices. An entrepreneur with characteristic p^* in cohort n at t is indifferent between participating in the pooling contract or taking a truth telling one. Equation (28) in Proposition 6 shows the rms labor demand for all cohorts. Under a pooling contract $\bar{p}_{nt}$=$\bar{p}_{nt}^{Pool}$ while under a truth telling contract $\bar{p}_{nt}$= p. In equilibrium, an entrepreneur taking a truth telling has a success probability of $p \leq p_{nt}^* <\bar{p}_{nt}^{Pool}$. Thus, each rm demand for labor is not continues in prices since for a type p^*_{nt} that is indifferent between one type of contract or the other, a small change in prices will make it switch to the other type of contract. Also note that this is the only source of discontinuity, since the entrepreneurs only participate in either of this two types of contracts by Proposition 8 and since labor demand is well behaved when the entrepreneurs type is different from p^*_{nt}.

From the individual demand (and supply) functions we construct the aggregate demand by computing the mass of rms taking truth telling contract and the mass taking a pooling contract for each cohort. These individual demands are locally continues functions of prices for every type but type p^* in each of this cohort. Nonetheless these types have zero mass in the cohort, implying that demand for the whole cohort is globally continues in the prices space since for all prices there is at most a type with mass zero whose demand is discontinuous being the everybody else s demand continuous in the same cohort. Aggregate demand accounted as the sum of each cohort demand is continuous and nite for every positive price by assumptions on $f(p)$.

Finally since aggregate excess labor demand and non-tradeable output supply are well behaved we conclude that equilibrium exist. ■

Chapter 2

Income Distriburtion as a Pattern of Trade

2.1 Introduction

Does income distribution matter for trade? I argue that it does. A two sector overlapping generation economy model is analyzed where one of the sectors is characterized by an imperfection in credit markets due to moral hazard. All effects appear on the supply side of the economy since agents preferences are specially chosen to avoid dealing with demand side effects of income inequality. I show that two economies with otherwise equal characteristics but with different income distribution will exhibit dissimilar comparative advantages in trade.

In this world, generations are linked by dynasties and parents leave bequest to their children, feature that introduces persistence in the distribution of wealth implying that takes time for the economy to converge to steady state income distribution and production levels. For that reason I also analyze the dynamics of wealth distribution to show that the economy is likely to pass through different phases of trade patterns in its development process. At initial stages of development, the model economy exhibits a comparative advantage over the sector characterized by no -or less- nancial frictions, to eventually revert its trade pattern at more advanced stages.

Rajan and Zingales (1996) show in a study for a large number of countries that those industrial sectors which need less external nance grow disproportionately faster in countries with less developed nancial markets.[1] Although their empirical study focus on the role of nancial development in economic growth, it is also evidence that industrial sectors with dissimilar needs for external nance exhibit different performance over time. This empirical fact suggests that income inequality can affect the sector s performance since in many types of credit market imperfections the entrepreneur s wealth sizes the amount of debt borrowed from banks.

In this article I show how this mechanism works. The distribution of income affects the amount invested in each sector, and with that, the country s comparative advantage to export the good produced in those sectors. Moreover, I show that when the amount invested in production depends nonlinearly on the entrepreneurs

[1] They de ne industry s need for external nance by the difference between investment and internal cash ow from data on US rms.

wealth two countries with same average endowment (per capita income), technology and preferences can exhibit dissimilar comparative advantages in trade only due to differences in income inequality. Societies with more equally distributed wealth will exhibit a comparative advantage in the sector needing more external nance.

A simple application of this model can be given by a two sector economy that produces agricultural and industrial products. The agricultural sector usually exhibits an advantage over industrial sectors regarding the access to efficient nancial contracts: land, which is the main factor of production in agriculture, serves as an excellent asset to put down as collateral in credit contracts.[2] On the contrary, some industrial sectors are usually populated by small rms, many of which are family rms managed by their owners. In these cases, the entrepreneurs personal wealth is usually a binding constraint at the time to borrow from banks.

To the best of my knowledge, the closest theoretical references to the role of income distribution as a pattern of trade come from the development area. Most of these articles debate on the relationship between credit market imperfections and economic growth, an idea stated by Schumpeter back in 1911.[3] Others, related to the rst group by the use of imperfect credit markets, focus on the stages of economic development followed by closed economies. Among the last group of papers I acknowledge

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[2] Farmers don t necessarily need to own all the land they use in production. They only need to own enough to contract the credit necessary for their scale of production.

[3] See Greenwood and Jovanovic (1990) and King and Levine (1993) and Rajan and Zingales (1996)

Lloyd-Ellis and Bernhardt (2000). They present a closed economy OLG model where agents leave bequest to their successors to resemble the Kuznet s hypothesis and other macroeconomic regularities. They introduce an imperfection in credit markets due to moral hazard where entrepreneurs with different managerial abilities might abscond part of their proceeds, although there is some utility penalty if the entrepreneur is apprehended (which occurs with positive probability). The entrepreneur s wealth linearly determines the amount loanable, which implies that income inequality does not have any impact on aggregate credit.[4] In equilibrium there is no default, a trade off of the simple set up on the credit market imperfection of their work. This feature differs from the credit market imperfection introduced in my model because here default occurs all the time. Moreover, the mass of entrepreneurs that default decreases over the development process under plausible initial conditions.

The rest of the paper is divided in four sections. In Section 2 I present a description of the environment. In Section 3 I show that under no imperfections in credit markets income distribution is irrelevant both for production and trade. In Section 4 I analyzed the economy when credit contracts can be imperfectly enforced, contrasting the results of Section 3. In subsection 4.1 I study income distribution and trade pattern dynamics. Lastly, in Section 5 I present some nal remarks and conclusions. Proofs can be found in the Appendix.

[4] Although it does have an effect on aggregate output due to the fact that technology exhibit decreasing returns.

2.2 Environment

I consider a small open economy populated by an in nite sequence of two period lived overlapping generations and with two productive sectors, A and B. At each period t a mass one of young agents is born. Youngsters become old at $t+1$, where they reproduce at a one to one rate, implying no population growth. Each agent j is endowed with $l_j \in [0, \bar{l}]$ units of labor when young, where l_j is an $i.i.d.$ random variable across generations and is drawn from a distribution $h(l_j)$ with mean 1 and cumulative distribution $H(l_j)$. This randomness resembles differences in wage income across the population and allows the asymptotic distribution of income to have some degree of inequality for all possible parameter values. When old, each agent can become an entrepreneur with probability μ or a worker endowed with one unit of labor with probability $1-\mu$. Labor is contracted at the beginning of each period. Entrepreneurs are the only type of agents capable of combining inputs to produce output in the A sector.

Technology in this sector is assumed to be given by

$$y_t^A = \begin{cases} F^A(k_t^A, l_t^A, 1) & \text{w/prob } e \\ 0 & \text{o.w.} \end{cases} \tag{2.1}$$

where $F^A(k_t^A, l_t^A, 1) = (k_t^A)^\alpha (l_t^B)^\beta$ and $\alpha+\beta < 1$ -one indivisible unit of entrepreneurial ability is required for production.[5] Capital and labor inputs are required at the beginning of the period while output takes place at the end of it. Also, output is subject to an idiosyncratic risk inherent to each project, with two possible realizations: successful or unsuccessful . The probability of the project being successful is given by e which is the effort level invested by the entrepreneur into managing: production or marketing activities. By assumption the effort is unobservable (or no enforceable) for all agents in the economy other than the manager, introducing a credit market failure that is studied below.

Technology in sector B is given by a standard constant return to scale production function that depends on capital and labor. Thus

$$y_t^B = F^B(k_t^B, l_t^B) \tag{2.2}$$

Timing of production is the same than in the A sector and capital and labor inputs across sectors are assumed to be perfect substitutes. This sector does not require a manager, or is less managing intensive, compared to technology in the A sector and there is no -less- incentive issues regarding production. This feature implies that rms in sector B are able to borrow from banks at the international interest rate since credit contracts are efficient. For concreteness, it is useful to associate sector A to an industrial sector where technology determines a small or medium rm size in

[5] More general production functions in capital and labor can be easily adapted to the analysis.

equilibrium.[6] On the contrary, sector B can be associated to a sector characterized by rms where production takes place at a large scale.[7] The relative small size of individual loans that takes place in sector A makes monitoring an inefficient task in credit contracts, something that doesn t happen at the B sector where the large scale of borrowing makes monitoring optimal.

Generations are linked as dynasties, and preferences do not only depend on their own consumption but also on bequest b left to their successors, independently of becoming an entrepreneur or not. This feature introduces persistence in a way that becomes clear below.[8] For analytical simplicity I assume that agents consume only when they become old. Their preferences are given by

$$E[U_t] = s(c_t^A)^\gamma (c_t^B)^\psi b_t^{1-\gamma-\psi} - \frac{a}{n+1} e_t^{n+1} \tag{2.3}$$

where s is a positive parameter useful for normalization purposes.

Utility is assumed to be homogeneous of degree one in consumption and bequest and a and n are positive parameters showing that utility decreases with effort input

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[6] The autopart industry is an example of this sector.

[7] Say, the steel industry.

[8] Introducing in nitely lived agents would have the same feature regarding persistence, although it would make the conclusions on pattern of trade less clear. In a model with in nitely lived agents intertemporal trade arises, contaminating the conclusion on comparative advantage with the intertemporal component of trade.

in production.[9] This assumption give us an indirect utility function that is linear in income, thus avoiding dealing with risk sharing issues when analyzing the role of income distribution and credit market imperfections on trade.

In what follows I assume that goods A and B are both tradables, that good B is the *numeraire* and that the bequest is normalized in terms of good B. Also, I assume that all markets are perfectly integrated with the rest of the world including capital markets and that labor is immobile across countries (although this assumption might not be always necessary).

Finally, the initial distribution of bequests across agents j are given by $G_0(b_j)$, $b_j \in [0, \bar{b}_0]$.

2.3 First best: Perfect enforcement

In this section I solve the model under the assumption that effort is observable and perfectly enforceable. When this is the case, I show that income distribution is irrelevant both for production and trade.

Since goods A and B are assumed to be tradable and this is a small economy, prices are determined in the rest of the world. The (relative) price of good A is denoted as P^A and assumed to be constant over time. At each period t total labor endowment in the economy is given by $L = 2 - \mu$, since there is a mass one of young agents with

[9] Further restrictive assumptions on this parameters are introduced later.

an average labor endowment of 1 unit plus a fraction $1-\mu$ of the old generation also with a unit labor endowment. Given technology in sector B, and assuming free entree conditions, equilibrium in this industry requires

$$\pi_t^B(w_t, r_t) = 0 \tag{2.4}$$

where π_t^B is the indirect pro t function in the B sector. Since the interest rate is given from the rest of the world, wages in this sector are determined by it. Moreover, since labor is perfectly substitutable across sectors, the wage rate in the economy is determined by the international interest rate. Assuming that the interest rate doesn t uctuate over time, wages will be constant (w).

The agents problem works as follows. At the beginning of each period all old agents know whether they are entrepreneurs or not, and they count on a certain wealth inherited plus labor income from youth. If they don t become entrepreneurs they supply their labor endowment inelastically and invest their savings in the bank at the rate r for the period. If they do become entrepreneurs they decided how much to invest within the rm and how much to save in the bank at the rate r. Finally at the end of the period they consume goods A and B and leave some bequest. Their demand functions are given by,

$$c_j^A = \gamma W_j$$

$$
\begin{aligned}
c_j^B &= \psi W_j \\
b_j &= (1-\gamma-\psi)W_j
\end{aligned}
\tag{2.5}
$$

where W_j is the agent j s wealth, determined by inheritance, labor income from youth and project outcome or labor income if they became entrepreneurs or not.

Let b^j be the amount inherited by an agent j. Then if the agent doesn t become an entrepreneur, his wealth at the moment of consumption is given by

$$
W_j = \left(b^j + w[l_j(1+r)+1]\right)(1+r) \tag{2.6}
$$

Matters are different when agents become entrepreneurs. Whatever the wealth is at the end of each agent adulthood, individual demands for consumption and bequest is still given by (5). By plugging these expressions into the utility function we obtain

$$
E_W[U_j] = E_W(W_j) - \frac{a}{n+1}e^{n+1} \tag{2.7}
$$

where I let $s^{-1} \equiv (\frac{\gamma}{P^A})^{\gamma}\psi^{\psi}(1-\gamma-\psi)^{1-\gamma-\psi}$. Note that expected utility depends only on expected end of period wealth and not on any other moments of its distribution due to the homogeneity assumption on preferences.

Let $N_j = b^j + (1+r)wl_j$ be the entrepreneurs net worth at the end of youth, given by labor endowment and wealth inherited[10]. Then, the entrepreneur decides how to distribute savings between his own rm and banks. To determine that, I rst compute the entrepreneurs problem assuming he can only save by investing in the rm. Therefore, I compute the entrepreneur indirect utility as a function of the amount invested within the rm N_j (the rm s net worth).

Let d be the amount borrow from the bank and i the interest rate on the debt contract. Under perfect enforcement, both lending interest rates and the amount borrowed can be made functions of net worth and effort. Optimal nancial contracts in this case can be derived from the following problem, [11]

$$\max_{k^A, l^A, e, i, d} EU = e\left[P^A (k^A)^\alpha (l^A)^\beta - [1 + i(e, N_j)] d(e, N_j)\right] - \frac{a}{n+1} e^{n+1} \tag{2.8}$$

subject to

$$e[1 + i(e, N_j)] d(e, N_j) - (1 + r) d(e, N_j) \geq 0 \tag{2.9}$$

$$r_k k^A + w l^A \leq d(e, N_j) + N_j \tag{2.10}$$

where r_k is the rental rate of capital assumed to be the same across sectors to simplify notation. The objective function is given by expected return, given by the probability

[10] For the purpose of this paper, two agents with different inheritance but with same wealth at the end of their youth are assumed to be indistinguishable.

[11] I drop time subscript for convinience.

of the project being successful times the proceeds of the firm after paying back debts, minus desutility of the effort exerted by the entrepreneur in his own project. Equation (9) is the bank s participation constraint. Financial contracts have to satisfy the non-negative profit condition for banks. Equation (10) is the firm s resource constraint: total expenditure on inputs should be financed by either net worth or debt.

From Equation (9) we obtain that $(1+i) = \frac{(1+r)}{e}$.[12] Because the lending interest rate can be contracted as a function of the entrepreneurs effort, we can make use of this expression to get rid off the lending rate in the objective function. This equation and Equation (10) allows to restate the problem as follows:

$$\max_{k^A, l^A, e} EU = eP^A(k^A)^\alpha (l^A)^\beta - (1+r)(r_k k^A + w l^A) + (1+r)N_j - \frac{a}{n+1}e^{n+1}$$

From the first order conditions of this problem we obtain some conditions that are worth analyzing. While a closed form solution exist, it is worth presenting the following optimality conditions to understand further assumptions on parameters.

$$k^{A*} = \left[\frac{P^A}{(1+r)}\left(\frac{\alpha}{r_k}\right)^{1-\beta}\left(\frac{\beta}{w}\right)^{\beta}\right]^{\frac{1}{1-\alpha-\beta}} e^{*\frac{1}{1-\alpha-\beta}} \tag{2.11}$$

$$l^{A*} = \left[\frac{P^A}{(1+r)}\left(\frac{\alpha}{r_k}\right)^{\alpha}\left(\frac{\beta}{w}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha-\beta}} e^{*\frac{1}{1-\alpha-\beta}} \tag{2.12}$$

[12] The financial contract is riskless for banks since they are able to completely diversify idiosyncratic risks.

$$ae^{*n} = \left[\frac{P^A}{(1+r)^{\alpha+\beta}}\left(\frac{\alpha}{r_k}\right)^{\alpha}\left(\frac{\beta}{w}\right)^{\beta}\right]^{\frac{1}{1-\alpha-\beta}} e^{*\frac{\alpha+\beta}{1-\alpha-\beta}} \tag{2.13}$$

In order for the solution to be interior the marginal revenue out of effort should be steeper than the marginal cost. This means that second order conditions for a maximum are satis ed if and only if $n \geq \frac{\alpha+\beta}{1-\alpha-\beta}$. Furthermore I will assume that no entrepreneur has incentives to put a level of effort that produces successful projects all the time. This means that $e^* < 1$ or $\frac{1}{a}\left[\frac{P^A}{(1+r)^{\alpha+\beta}}\left(\frac{\alpha}{r_k}\right)^{\alpha}\left(\frac{\beta}{w}\right)^{\beta}\right]^{\frac{1}{1-\alpha-\beta}} < 1$. In what follows I let

$$z \equiv \left[\frac{P^A}{(1+r)}\left(\frac{\alpha}{r_k}\right)^{\alpha}\left(\frac{\beta}{w}\right)^{\beta}\right]^{\frac{1}{1-\alpha-\beta}} \tag{2.14}$$

and I assume that the two conditions on parameters always hold.

Financial contracts also have closed form solutions given by

$$i^* = \frac{1+r}{e^*} - 1 \tag{2.15}$$

$$d^* = \max\left[(\alpha+\beta)ae^{*n+1} - N_j, 0\right] \tag{2.16}$$

where the lending rate is only function of the effort, while the amount lend is a function of effort (and returns to scale) and net worth. If net worth is big enough to cover for the whole cost of (rst best) production, borrowing is not needed.

The indirect expected utility function for the entrepreneur that invests an amount

N_j is given by

$$U^{FB*} = \frac{e^{*n+1}a(1-\alpha-\beta)}{(n+1)}\left[n - \frac{\alpha+\beta}{1-\alpha-\beta}\right] + (1+r)N_j \tag{2.17}$$

which is positive by assumption even in the case where $N_j = 0$.[13] The entrepreneurs indirect expected utility is given by the sum of net utility gains from being an entrepreneur plus income coming from inheritance and labor income during youth.

The main result of this section, obtained by other scholars in different frictionless contexts, is summarized in the following proposition.[14]

Proposition 1 *Under perfect enforcement, the rms scale of production (given by capital and labor inputs) is independent of the entrepreneur s net worth.*

The proof follows immediately by observing that the solution for labor and capital comes from Equations (11) to (13) being them independent of N_j.

With this result at hand, the aggregate demand for capital and labor in sector A are given by

$$\begin{aligned} K_t^A &= \mu k_t^{A*} \\ L_t^A &= \mu l_t^{A*} \end{aligned} \tag{2.18}$$

[13] Superscript FB stands for rst best.

[14] See Modigliani and Miller and others.

Plugging Equation (13) into (12) gives

$$l_t^{A*} = \frac{\beta a e^{*n+1}}{w(1+r)} \tag{2.19}$$

Also, aggregate labor supply is given by $(2-\mu)$. Wages are going to be determined in sector B as long as excess labor demand in sector A at the wage rate given by sector B is nonpositive ($L_t^A \leq 2-\mu$) or

$$\frac{\beta a e^{*n+1}}{w(1+r)} \leq \frac{2-\mu}{\mu} \tag{2.20}$$

This conditions implies that labor endowment is big enough to have both sectors A and B active even under perfect enforceability. For the rest of the sections analyzed below, this constraint can be relax since it will be shown that under imperfect contracting this sector will underinvest.

It is important to note that by Equations (5)-(7) aggregate demand for consumption of goods A and B are independent of income inequality -given by higher moments of the income distribution determined by $G(b_j)$ and $H(l_j)$-, since individual demand functions are linear in wealth. From this result I derive the following statement.

Proposition 2 *Under perfect enforcement, comparative advantage of a small open economy like the one described above is independent of both per capita income and income inequality.*

The proof follows from Proposition 1 and the linear relationship between aggregate consumptions for good A and B and wealth. Thus under perfect enforcement, two similar small open economies -in the sense of preferences, aggregate labor endowment and technology as those described above- will have the same comparative advantage regardless of their income distribution. I come back to this point later.

2.3.1 Income distribution dynamics under perfect enforcement

Although under perfect enforcement income distribution is irrelevant as a pattern of trade, I described the evolution of such distribution over time because it helps for comparison with the case of imperfect credit markets.

Given an initial distribution of bequests among members of the young population, we are able to compute next period distribution of both bequest and wealth for all successive generations.[15] From Equation (32), an entrepreneur with wealth $N_j < N^*$ gets an actual income of

$$I_j = \begin{cases} (1+r)ze^{*\frac{\alpha+\beta}{1-\alpha-\beta}}\left[1-(\alpha+\beta)e^*\right]+(1+r)N_j & if\ succesfull \\ (1+r)N_j & o.w. \end{cases} \tag{2.21}$$

[15] Each entrepreneur has the option to fully nance his project with borrowing and diversify completely the risk or only nance externally what cannot fund by himself (or any combination of this two alternatives). Due to the homogeneity on preferences, entrepreneurs are indifferent between these options. I assume that when this is the case the entrepreneur chooses to fully fund the project by borrowing (which would be the case if preferences have some degree of risk aversion).

Furthermore, income for an agent that becomes a worker when old is given by $I_j = (1+r)(N_j + w)$. Let $\phi \equiv (1-\gamma-\psi)(1+r) < 1$. Thus, the law of motion for bequests under rst best is determined as follows

$$b_j = \begin{cases} \phi \left\{ z e^{*\frac{\alpha+\beta}{1-\alpha-\beta}} \left[1-(\alpha+\beta)e^*\right] + N_j \right\} & \text{w.prob } e^*\mu \\ \phi N_j & \text{w.prob } (1-e^*)\mu \\ \phi(N_j + w) & \text{w.prob } 1-\mu \end{cases}$$

where $N_j = b + (1+r)lw$, where b is the amount inherited by the agent.

From this law of motion and any arbitrary distribution of bequests, we are able to determined the path followed by the income distribution over time. This is used only for comparison with the case under imperfect enforcement. In what follows I assume

$$(1-\gamma-\psi)(1+r) < 1 \tag{2.22}$$

For future references, we state the results regarding income distribution with the following proposition.

Proposition 3 *Under perfect enforcement, there is a unique stationary, ergodic distribution of bequest* $G_\infty^{FB}(b)$ *and wealth* $F_\infty^{FB}(N)$ *for the economy described above, with* $b \in [0, \bar{b}^{FB}]$ *and* $N \in [0, \bar{b}^{FB} + (1+r)w\,\bar{l}]$*, where*

$$\bar{b}^{FB} = \frac{\phi}{1-\phi} \left\{ z e^{*\frac{\alpha+\beta}{1-\alpha-\beta}} \left[1-(\alpha+\beta)e^*\right] + (1+r)w\,\bar{l} \right\} \tag{2.23}$$

Proof. See Appendix.

In the next section I describe the behavior of the economy when credit markets are imperfect and I compare the distributions of wealth under these two cases.

2.4 Second best: Imperfect enforcement

The problem under imperfect enforcement is slightly different, although the results change drastically. In this case, nancial contracts cannot be written as functions of the effort level since effort is not observable for any agent in the economy other than the entrepreneur undertaking the project. As I show below, this assumption will generate some surprising changes in the way sectors in the economy perform over time.

By letting the entrepreneur s wealth N_j be small enough we allow rms to seek for external nance. In this case, the entrepreneur s problem becomes

$$\max_{k^A, l^A, e} EU = e\left[P^A(k^A)^\alpha(l^A)^\beta - [1+i(N_j)]d(N_j)\right] - \frac{a}{n+1}e^{n+1} \tag{2.24}$$

subject to

$$\begin{aligned} r_k k^A + w l^A &\leq d(N_j) + N_j && (2.25) \\ & i(N_j), d(N_j) \qquad \textit{given} && (2.26) \end{aligned}$$

where nancial contracts are given for the entrepreneur and only functions of his wealth.

The rst order conditions of this problem give us[16]

$$\alpha P^A (k^A)^{\alpha-1}(l^A)^{\beta} = r_k(1+i) \tag{2.27}$$

$$\beta P^A (k^A)^{\alpha}(l^A)^{\beta-1} = w(1+i) \tag{2.28}$$

$$P^A (k^A)^{\alpha}(l^A)^{\beta} - [1+i(N_j)]d(N_j) = ae^n \tag{2.29}$$

Solving for capital and labor and plugging the result in Equation (29) we obtain

$$(1-\alpha-\beta)\left[\frac{P^A}{(1+i)^{\alpha+\beta}}\left(\frac{\alpha}{r_k}\right)^{\alpha}\left(\frac{\beta}{w}\right)^{\beta}\right]^{\frac{1}{1-\alpha-\beta}} + (1+i)N_j = ae^n \tag{2.30}$$

Now equilibrium conditions in the banking sector implies that $1+i = \frac{1+r}{\hat{e}}$, where $\hat{e}$ is the bank s expectation of the entrepreneur s effort level exerted under that contract, also a function of his wealth. Under rational expectation $\hat{e} = e$. Plugging this two conditions into Equation (30) give us the following optimality condition for effort under imperfect enforcement,

$$(1-\alpha-\beta)(1+r)ze^{\frac{\alpha+\beta}{1-\alpha-\beta}} + \frac{(1+r)N_j}{e} = ae^n \tag{2.31}$$

[16] Actually from the FOC we derive two solutions, but I norrowed it down to the one corresponding to a maximum of the problem.

where z is a function of parameters as stated in (14). This expression implicitly de nes and optimal effort level as a function of the entrepreneurs net worth $(e(N_j))$. Solving for e allows to work backwards and solve for the rest of the variables, nancial and non nancial ones. Equation (31) give us the rst result of this section. Note that the optimal level of effort invested by the entrepreneur is now dependent on the entrepreneurs net worth. This is necessarily inefficient because all entrepreneurs have the same managerial ability regardless of their income endowment.

With the help of Figure 1 I show the implicit relation between the entrepreneur s effort and his wealth.

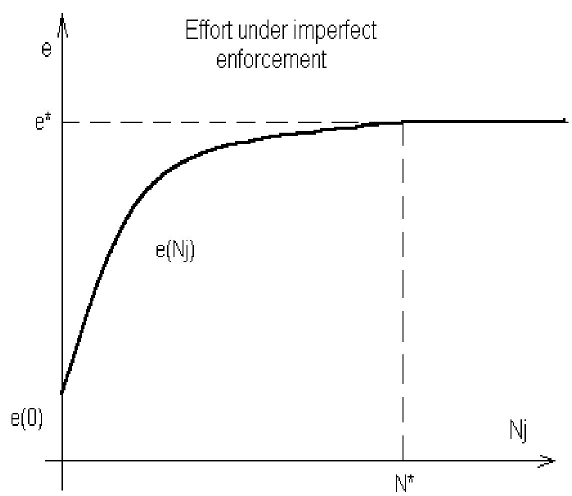

Figure 2.1: Effort under imperfect enforcement

Letting $N_j = 0$ and ruling out $e = 0$ because it is not an arbitrage free competitive solution of our nancial contract, we are able to obtain the correspondingly effort level.

$$e(0) = \left[\frac{(1+r)z(1-\alpha-\beta)}{a}\right]^{\frac{(1-\alpha-\beta)}{n-(n+1)(\alpha+\beta)}} = (1-\alpha-\beta)^{\frac{(1-\alpha-\beta)}{n-(n+1)(\alpha+\beta)}} e^* \tag{2.32}$$

where the exponent is positive under previous assumption. Expression (32) shows that under no net worth, the effort level exerted by the entrepreneur is a proportion (<1) of the efficient effort level e^*. Under reasonable parameter values the difference between this two effort level can be signi cant.

Differentiating the optimality condition for effort and using it into the differentiated equation give us

$$\frac{de}{dN_j} = \frac{e}{\left[[n(1-\alpha-\beta)-(\alpha+\beta)]ze^{\frac{1}{1-\alpha-\beta}} + (n+1)N_j\right]} > 0 \tag{2.33}$$

which is positive under previous assumptions. Thus, another result is that entrepreneurs effort is increasing in their wealth. The reason for that is that as the amount invested in the project by the entrepreneur increases, the con ict of interests between entrepreneurs and banks is reduced. This happens because the incentives of entrepreneurs to shirk are reduced as the proportion self nanced increases. Interestingly, when the entrepreneur has all the money to fully nance the project the agency costs disappear since there is no con ict of interest because there is no borrowing. To see this, note that when $N_j = wl^{A*} + r_k k^{A*}$ or the entrepreneurs wealth is big enough to nance the efficient scale of operations obtained in Section 3, $N^* = (\alpha + \beta)ze^{\frac{1}{1-\alpha-\beta}}$. Plugging this net worth level into the optimality condition for effort under imperfect enforceability, give us the rst best solution for effort e^*. If the entrepreneur wealth is bigger than N^* then the optimal investment plan would be to invest N^* in the rm and put the rest in the bank at the interest rate r.

Another important result for what follows is that from Equation (33) we see that the optimal effort level is a concave function of the entrepreneur s wealth. Before going further I summarize the results obtained in the following proposition.

Proposition 4 *Under imperfect enforcement, the optimal effort level exerted by the entrepreneur, $e(N_j)$, is an increasing and concave function of his net worth N_j and is bounded below by $e(0)$ and above by the efficient effort level, e^*, implying that the optimal scale of operations depends on the entrepreneurs wealth. Moreover, there is aggregate underinvestment in sector A.*

Obtaining the optimal effort $e(N_j)$ allows to solve for the nancial contracts by plugging this effort level into Equations (15) and (16).

The indirect expected utility function of an entrepreneur with wealth N_j is given by

$$U^*(N_j) = (1 - \alpha - \beta)(1 + r)ze^{\frac{1}{1-\alpha-\beta}} + (1 + r)N_j - \frac{a}{n+1}e^{n+1} \qquad (2.34)$$

Using the optimality condition for effort and assuming $N_j < N^*$ gives

$$\begin{aligned} U^*(N_j) &= \frac{n}{n+1}\left[(1-\alpha-\beta)ze(N_j)^{\frac{1}{1-\alpha-\beta}} + (1+r)N_j\right] \\ &= \frac{n}{n+1}ae(N_j)^{n+1} \end{aligned} \tag{2.35}$$

First, note that from expression (35) we obtain that the indirect expected utility is the sum of utility from income minus desutility from effort. Also, from Proposition 4 we know that effort is an increasing function of wealth as long as $N_j < N^*$. This implies that the expected return on net worth is higher than the interest rate whenever wealth is not enough to cover the cost at the efficient scale of operations. A comparison between this expression with (17) shows that while under perfect contracting the marginal return on net worth is constant and equal to $(1+r)$, under no enforceability the marginal return on net worth is higher (for the relevant interval). The reason for this is that higher net worth helps to support better incentive compatible contracts through higher effort levels.

Secondly, note that the utility is an increasing function of effort for all $e \in [e(0), e^*]$ as $n > 0$. This shows that the indirect utility level is higher than $(1+r)N_j$ and that entrepreneurship is pro table.

Finally, from the last part of expression (36) we con rm that the indirect expected utility is lower (equal) than its rst best for net worth levels below (equal) N^*.

Aggregation in the case of imperfect credit markets is also possible, although less trivial since now each entrepreneurs output will depend on the their own wealth. Figure 2 displays the relation between an individual rm s equilibrium output level and its owner wealth.

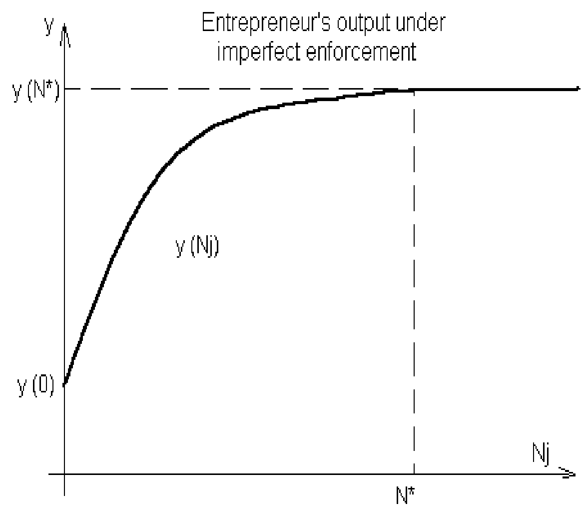

Figure 2.2: Firms output under imperfect enforcement

The levels $y^A(N^*)$ and $y^A(0)$ correspond to the incentive compatible entrepreneurs output levels, contingent on being successful, for the extreme cases where their wealth is N^* and 0 respectively. Note that when the entrepreneurs wealth is enough to fully nance their own projects, the output level is efficient. Also note that, in general, there is underinvestment in this sector. This function is

$$y^A = (1+r)ze(N_j)^{\frac{\alpha+\beta}{1-\alpha-\beta}}$$

Note that only a fraction $e(N_j)$ of them won t fail. Taking this into account, aggregate output in sector A is computed as,

$$Y^A = (1+r)\mu z \int_0^\infty e(N_j)^{\frac{1}{1-\alpha-\beta}} dF(N_j) \tag{2.36}$$

where

$$F(N_j) \equiv \int_0^{N_j} \int_0^x h\left(\frac{x-b}{w(1+r)}\right)\, dG(b)dx \tag{2.37}$$

and $N_j = b + lw(1+r)$.

It is convenient to rearrange the expression for aggregate output in sector A as follows

$$Y^A = \mu(1+r)zE_{N_j}\left[e(N_j)^{\frac{1}{1-\alpha-\beta}}\right] \tag{2.38}$$

Having stated aggregate output in Sector A like in Equation (38) we can now study the role of income distribution over production in this sector. First, let $Q(N_j) = e(N_j)^{\frac{1}{1-\alpha-\beta}}$. Then, after some tedious algebra it can be shown that

$$\frac{d^2Q(N_j)}{dN_j^2} = \frac{e^{\frac{\alpha+\beta}{1-\alpha-\beta}-1+n}(1+r)(n+1)a}{(1-\alpha-\beta)\left[(n+1)ae^n - (1+r)ze^{\frac{\alpha+\beta}{1-\alpha-\beta}}\right]^2}\frac{de}{dN}\left(\frac{\alpha+\beta}{1-\alpha-\beta} - n\right) \leq 0 \tag{2.39}$$

Because $Q(N_j)$ is a concave function of wealth we can conclude

Proposition 5 *Let $F_i(N)$ be the end of youth distribution of wealth of Country i. Under imperfect enforcement, if $F_2(N)$ rst order stochastically dominates $F_1(N)$ or $F_2(N)$ second order stochastically dominates $F_1(N)$, then $Y_2^A > Y_1^A$.*

Proof. See Appendix.

From this proposition it results that

Corollary 2 *Under imperfect enforcement, if country* 2 *has otherwise equal characteristics than country* 1 *but* $F_2(N)$ *second order stochastically dominates* $F_1(N)$*, then country* 2 *has a comparative advantage to produce good* A *over country* 1*.*

By Proposition 5 country 2 will produce more of good A. Moreover, it will produce less of good B because it is not only the case that entrepreneurs (on average) will have greater incentives to produce in country 2 over country 1, but also this country will allocate more labor into sector A than country 1 as the rst order conditions (27)-(29) show.[17]

It is worth comparing the cases of perfect and imperfect credit markets. While under perfect enforcement the comparative advantage to produce good A for two countries with otherwise equal characteristics but with different income distributions are the same -as stated in Proposition 2-, in the case of imperfect enforcement, two countries with different income distributions will exhibit dissimilar comparative advantages. The country with a dominant income distribution in the second order sense

[17] Solving for individual labor demand as a function of effort gives

$$l^A = \left[\frac{P^A}{1+r}\left(\frac{\alpha}{r_k}\right)^{\alpha}\left(\frac{\beta}{w}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha-\beta}} e(N)^{\frac{1}{1-\alpha-\beta}}$$

Aggregating accross entrepreneurs and applying Theorem 1 to this expression shows that the country exhibiting second order stochastic dominance will allocate more labor in sector A.

ill export (import) more (less) of the A good. Moreover, a country that suffers from an adverse shock that increases its income inequality in the second order sense keeping its per capita income the same, will reduce their comparative advantage in sector A and might end up reverting their pattern of trade from the bank intensive sector to the non intensive one.[18] The same reasoning applies to two countries with otherwise equal characteristics (including per capita income) but with different income distribution in the second order sense.

Because the neutrality of the income distribution over the pattern of trade is broken under imperfect credit markets, income distribution dynamics are important for the dynamics of trade pattern. Next, I analyzed the former one and its link to

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[18] Such an economy will exhibit a worse Gini Coe cient. See that a better Gini Coe cient implies a bigger area under the Lorentz curve. Let L_i be such area when the distribution of wealth is given by $F_i(N)$.

$$L_i = \frac{\int_0^{\bar{N}} \int_0^{N} x dF_i(x) dF_i(N)}{E_{F_i}(N)}$$

Integrating by parts twice and assuming mean preserving we get

$$2E(N)\left[L_1 - L_2\right] = \int_0^{\bar{N}} I(F1 > F2)\left[F_1(x)^2 - F_2(x)^2\right] dx$$

where the indicator function takes values one or minus one.

Since $F_1^2 - F_2^2 = (F_1 - F_2)(F_1 + F_2)$, this expression can be integrated by parts once again with the following result

$$2E(N)\left[L_1 - L_2\right] = \int_0^{\bar{N}} \int_0^{N} I(.)\left[F_2(x) - F_1(x)\right] dx \; d\left[F_1(N) + F_2(N)\right]$$

since mean preserving implies $\int_0^{\bar{N}} I(.)\left[F_2(x) - F_1(x)\right] dx = 0$. Because F_2 dominates F_1 in the second order sense, then $L_2 > L_1$ and the Gini Coe cient for country 2 is better than under country 1.

the dynamics of trade pattern.

2.4.1 Income distribution dynamics under imperfect enforcement

In this section I describe the law of motion for bequest and wealth among agents of the same generation. In this case, an entrepreneur with wealth $N_j < N^*$ gets an actual income of

$$I_j = \begin{cases} (1-\alpha-\beta)(1+r)ze(N_j)^{\frac{\alpha+\beta}{1-\alpha-\beta}} + \frac{(1+r)N_j}{e(N_j)} & if\ successful \\ 0 & o.w. \end{cases} \tag{2.40}$$

since the entrepreneur will risk all his wealth in the investment project because the interest rate the bank is charging him is a decreasing function of the rms net worth.

Using the optimality condition (Equation (31)) gives

$$I_j = \begin{cases} ae(N_j)^n & if\ successful \\ 0 & o.w. \end{cases}$$

As before, income for an agent that becomes a worker when old is given by $I_j =$

$(1+r)(N_j+w)$. The law of motion for bequests under imperfect enforcement is

$$b_j = \begin{cases} \phi\left\{\frac{ae(N_j)^n}{1+r} + \max[N_j - N^*, 0]\right\} & \text{w.prob } e(N_j)\mu \\ \phi \max[N_j - N^*, 0] & \text{w.prob } (1-e^*)\mu \\ \phi(N_j + w) & \text{w.prob } 1-\mu \end{cases} \tag{2.41}$$

where again $N_j = b + (1+r)lw$.

Let

$$B^E(N) = \phi\left\{\frac{ae(N_j)^n}{1+r} + \max[N_j - N^*, 0]\right\} \tag{2.42}$$

$$B^W(N) = \phi(N_j + w) \tag{2.43}$$

where superscripts E and W stand for entrepreneur and worker. The law of motion for bequest and wealth can be better understand with the help of Figure (3). As in the rst best case, three curves describe the possible levels of bequest that an entrepreneur with wealth N can leave to his successor, $B^E(N)$, $B^W(N)$ and $\phi \max[N_j - N^*, 0]$. As before the worse scenario comes from being an unsuccessful entrepreneur. On the contrary, highest bequest occurs when an agent with wealth $N \geq N_0$ becomes a successful entrepreneur or an agent with wealth $N < N_0$ becomes a worker.[19] The 45 degree line describe the set of points for which $N = b$. Under imperfect enforcement,

[19] In fact, parameters values determine whether $e(0)$ is bigger or lower than ϕw.

the offspring of an entrepreneur with wealth $N > N_0$ can at most have an end of youth wealth of $B^E(N)+(1+r)w\,\bar{l}$. Thus, in the long run, all dynasties that initially started with a wealth bigger that $\bar{N}$ will run down wealth even if all successor get maximum labor endowment on youth $(\bar{l})$ and become entrepreneurs, in which case they end up at $\bar{N}$. Thus, in the long run, and independently of initial conditions, the wealth distribution is bounded above by $\bar{N}$ while the distribution of bequests is bounded above by $B^E(\bar{N})$ shown as $\bar{b}^{SB}$ in Figure (3).

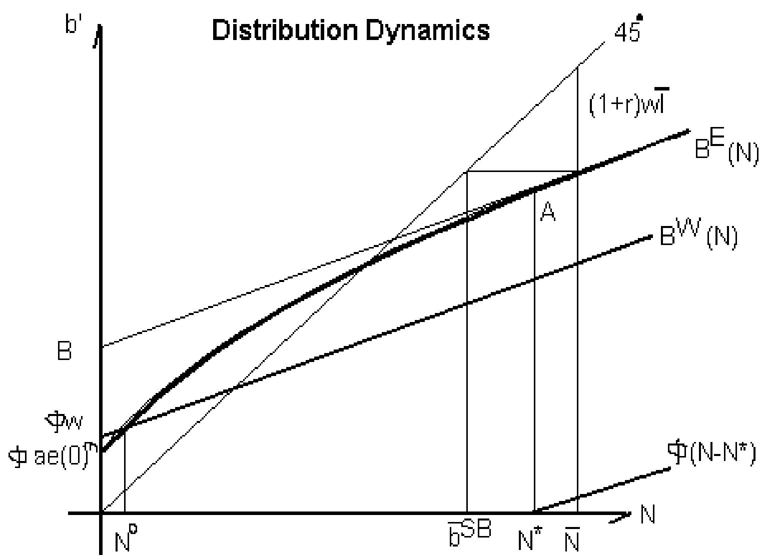

Figure 2.3: Bequest

By the same token, the wealth distribution has a lower bound of zero since the offspring of an unsuccessful entrepreneur with lowest youth labor endowment will have no end of period wealth. Given the assumptions on the initial distribution of bequests and youth labor endowment, we can compute the law of motion for the distribution of wealth for each generation. I summarize this result in the following statement.

Proposition 6 *Under imperfect enforcement, the law of motion for the end of youth distribution of bequest is given by*

$$G_{t+1}(b) = \begin{cases} \int_0^b v_t(x)dx & \forall b \in (0, \infty) \\ 1 - \int_{x\in(0,\infty)} v_t(x)dx & b = 0 \end{cases} \tag{2.44}$$

where v_t is de ned as

$$\begin{aligned} v_t(b) &= \mu e\left(B^{E-1}(b)\right) I(b > \phi ae(0)^n) \int_{B^{E-1}(b)-(1+r)w\bar{l}}^{B^{E-1}(b)} h\left(\frac{B^{E-1}(b) - x}{(1+r)w}\right) dG_t(x) + \\ &\quad \mu[1 - e^*] \int_{\frac{b+\phi N^* - \phi(1+r)w\bar{l}}{\phi}}^{\frac{b+\phi N^*}{\phi}} h\left(\frac{b + N^*\phi - x\phi}{\phi(1+r)w}\right) dG_t(x) + \\ (1-\mu)I(b &> \phi w) \int_{\frac{b-w\phi-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b-w\phi}{\phi}} h\left(\frac{b - \phi(x+w)}{(1+r)w\phi}\right) dG_t(x) \end{aligned} \tag{2.45}$$

Proof. See Appendix.

The distribution of wealth for every period can be directly inferred from this proposition. Because all agents with characteristics (l, b) satisfying $N = b + (1+r)wl$ have the same wealth at the end of their youth, and having the sequence of distributions for bequests across the population given by Proposition (7), the distribution of wealth for every period is determined using Expression (37).

A ot er es lt ha fo lo s f om he re io s p op si io is he xi te ce f a er od c d st ib ti n f r b th ea th nd eq es .

P op si io 7 *U de im er ec en or em nt th re s a ni ue ta io ar , e go ic is ri ut on f b qu st* $^{SB}(b$ *an we lt* F^S_∞ ( ) *f r t e e on my es ri ed bo e, it* $b \in$ 0, $^{-SB}$ *an* $N \in$ 0, $^{-SB}$ (1 r) $\bar{l}$] *wh re* $^{-SB}$ *nd* $^{-}$ *ar th x d p in s o th fo lo in ma pi g,* [0]

$$\bar{b}\ ^{B} = \ ^{E}\left[^{-SB} \quad (1 \quad r) \quad \bar{l}\right]$$

$$^{-} = B\ (\bar{N} \ + (\ + r\ w\ \bar{l}$$

Pr of Se Ap en ix

It s w rt no in th t u de pe fe t e fo ce en , t e l w o mo io fo be ue t (nd en e w al h) an e o se ve ro Fi ur (3 by et in B^E N) e e ua to he rs be t l ve of eq es le t b su ce sf l e tr pr ne rs se me t A). s c m- on en e s gg st th di tr bu io of eq es an we lt im ro es n t e st rd r s oc as ic en e u de pe fe t e fo ce en , b ca se he ay ff f r s cc ss ul nt e- re eu s i hi he (m re ffic en) u de th s c nd ti n.

he ac th t P op si io (8 pr ve th ex st nc of n a ym to ic is ri ut on f w al h a d b qu st s t me oe on sh ws ha th re s a ni ue on ru pa te n

[2] *Th er od c s ts f b qu st nd ea th re ot ec ss ri y c nv x.*

of trade for this economy. The dynamic trade pattern in the transition from any initial distribution of wealth (or bequest) is in general dependent on this initial conditions. Nevertheless, some general conclusion can be derived under some (plausible) initial conditions for the distribution of wealth or bequest. Let $\bar{Y}^A$ be the asymptotic aggregate output level in sector A (the one corresponding to the asymptotic wealth distribution).

Proposition 8 *Under imperfect enforcement, if for some t G_t rst order stochastically dominates G_{t-1}, then Y_t^A converges monotonically to $\bar{Y}^A$.*

Proof. See Appendix.

This proposition suggest that under plausible conditions, a small open economy like the one described before is likely to exhibit a pattern of trade where initially the country imports good A an eventually ends up exporting it (or importing less).

Figure 4 describes the pattern of trade dynamics under the initial conditions given in Proposition 9 in the presence of imperfect enforcement in nancial markets. The vertical axes represents aggregate consumption and production levels of good A while the horizontal axes represents the corresponding aggregate consumption and production levels of good B. The line $\overline{OC_0C}$ shows the income expansion path, which is a straight line because of the assumptions made on agents preferences. The curves represent the production possibility frontier corresponding to the initial period t and the asymptotic production possibility frontier for the limit as $t \to \infty$. Given the re at ve ri e o go d A as um d t be on ta t o er im , a gr ga e p od ct on t

t me is es ri ed y p in P_0 hi e a gr ga e c ns mp io by $_0$, nd ca in th t

t e e on my ta ts mp rt ng oo A a d e po ti g g od . A ti e p ss s t e

p od ct on os ib li y f on ie ex an s b ca se he e a e m re nt ep en ur wi h

h gh r w al h l ve s t at xe t m re ffo t i to an gi g i se to A. ll he ro

du ti n p ss bi it fr nt er sh re he am po nt $\bar{}$ co re po di g t th ca e w er

al la or s a lo at d i th s s ct r, in e t er is o p od ct ve ol fo we lt in ec

to B.[1] W il co su pt on xp nd th ou h t e i co e e pa si n p th pr du ti n

f ll ws pa h g ve by $\overline{{}_0P}$ Si ce ec or be om s m re nd or effi ie t, he

co om mo es es ur es ca it l a d l bo) f om ec or to ar s A Th s, hi e

t e e on my ta ts it a c mp ra iv ad an ag ov r s ct r B it tr de at er ca

be ve tu ll re er ed ow rd ec or .

T is ea ur of he od l a re s w th mp ri al nd ng by aj n a d Z ng le (1 96 wh re nd st ia se to s w th is im la ne ds nd ha ac er st cs ow rd x- er al na ce xh bi di er nt er or an e o er im . I th s m de , t e s ct r t at s c ar ct ri ed y i pe fe t c ed t a ce s e pe ie ce sl w p oc ss f d ve op en , w il th se to wi h p rf ct re it cc ss ta ts ig t a ay t i s l ng un te dy ta e p od ct ve ev l.

---

[1] T e s op of he ro uc io po si il ty ro ti rs re n it t t is oi t b ca se ec no og in ec or ex ib ts ec ea in re ur s t sc le

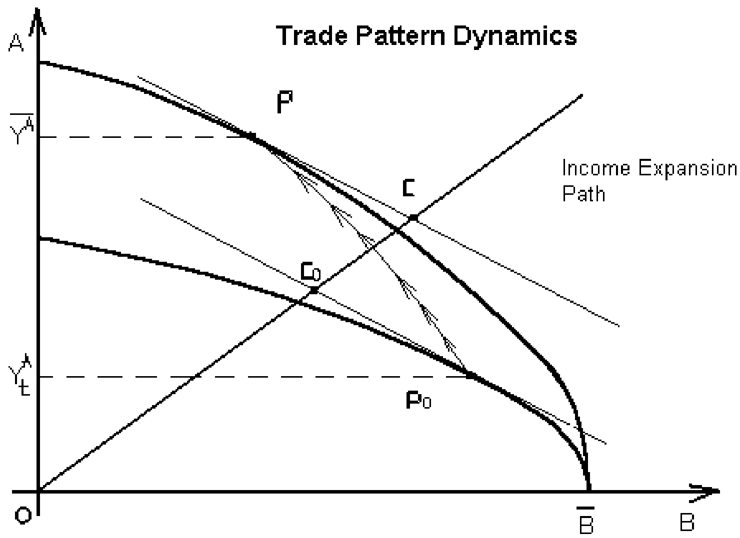

Figure 2.4: Trade pattern over time

Finally, it is interesting to notice that under perfect enforcement, the economy doesn t exhibit any dynamic trade pattern, and goes write away to a production-consumption combination that is even more extreme than $P - C$.

2.5 Final remarks and extensions

In summery, nancial frictions like the one described here are able to break the neutrality result regarding comparative advantage given in Proposition 2 in Section 2. Moreover, because agents care about their offsprings -by leaving them a fraction of their own wealth- the distribution of income will be history dependent in its transition to the long run steady state distribution. Hence both, nancial frictions and bequest can be responsible for dissimilar trade pattern dynamics exhibited by two otherwise equal economies that are at different stages of development.

The model describes a two sector economy with different characteristics. One sector has a technology that needs management as an indivisible factor of production and operates at a small scale and where the monitoring costs of management are too big compared to the total profitability of the enterprise, making financial contracts imperfect. The other is assumed to be a frictionless sector: some sectors operate at such a big scale that monitoring cost of efficient financial contracts are negligible compared to the total cost of production. The steel industry is an example of such sector: while financial contracts in these sectors are still imperfect, they are definitely superior contracts than those observed between banks and small firms. Some sectors are more liquid in the sense of Rajan and Zingales (1996). Others like the agricultural sector are able to collateralize loans with the same factors (land) utilized in production due to their high liquidation value.

Because the model has a frictionless sector that utilizes capital that freely moves between sectors and countries, labor wages are determined by the technology in this sector. While there are many frictionless sectors that use capital and labor, others -like the agricultural sectors- use land and labor. The dynamics presented in this paper work under the assumption that wages are constant over time. For that reason the agricultural sector should be interpreted as a subsistence sector, like in Lloyd-Ellis and Bernhardt (2000) or a sector with constant marginal product of labor.[22]

A natural extension of the present model would be to assume a constant returns to scale technology on land and labor in the agricultural sector. This model would have the attractiveness that not only labor migrates over time from this sector to the industrial one, but also that it displays increasing wages over time. In such case, the bequest function of successful entrepreneurs, given by curve $B^E(N)$ in Figure 1, would shift downwards over time while the corresponding one for workers -$B^W(N)$- would shift upwards. Such setup allows to study whether this economy might exhibit trade pattern traps. The fact that labor wages increases with a dominant distribution of wealth in the rst order sense, might result that at the same relative prices P^A given from abroad, two economies might nd different asymptotic wealth distribution and long run equilibrium wages, which indeed imply a trade and poverty trap. The trade off works as follows: the aggregate labor demand of Sector A depends negatively on wages and positively on (better in the rst order sense) asymptotic distribution of income. A sufficient condition for a trap to exist is that some aggregate labor demand level can be supported by a low wage and bad asymptotic distributions of wealth and by a high wage together with a good asymptotic distribution of wealth. Adapting equation (12) to the case where effort is unobservable shows aggregate labor depends negatively on wages directly, and indirectly through the effort exerted

[22] This technology would exhibit increasing returns in land and labor.

(since higher wages implies less pro ts and effort) and positively on entrepreneurs wealth distribution through effort. But higher wages, might imply that the asymptotic distribution of wealth is higher whenever μ is low enough (when entrepreneurs are relatively scarce). This indeed suggest that this two opposed forces in the trade off might outweigh each other at some positive aggregate labor demand in Sector A. The existence of such case would complement Boyd and Smith (1997) development traps arguments, since no credit rationing is needed here.

Finally, more standard arguments like that by Torvik (1993) can be adapted to study traps in trade. This paper shows that because individual talent and bequest are relevant in deciding whether or not to invest in education, countries with higher income will invest more in education and will have higher growth rates due to learning. If education is a complementary factor of production in one of the sectors and not in the other -like the agricultural one-, to push the argument to extremes, one could study wether trade traps can occur when these countries trade with the rest of the world.

2.6 Appendix

Proof of Proposition 3. First note that if there is an ergodic distribution of bequests, there is an ergodic distribution for wealth and viceversa. From the law of motion for bequests, $\frac{db_j}{dN_j} = (1-\gamma-\psi)(1+r) < 1$. Since the best outcome any agent could expect is given by the one corresponding to a successful entrepreneur, we look at the rst part of the expression for bequest to determine the upper bound of the ergodic set of this variable. By replacing the entrepreneurs upper level of wealth $N = \bar{b} + (1+r)w\ \bar{l}$ given by the luckiest agent with maximum level of bequest, we obtain the following condition for $\bar{b}$.

$$\bar{b} = (1-\gamma-\psi)\left\{(1+r)ze^{*\frac{\alpha+\beta}{1-\alpha-\beta}}\left[1-(\alpha+\beta)e^{*}\right]+(1+r)\left[\bar{b}+(1+r)w\ \bar{l}\right]\right\}$$

From which $\bar{b}$ can be derived.

Considering that the worse thing that can happen to a dynasty with wealth N_j is to become an unsuccessful entrepreneur for many periods and get no labor endowment in all these periods, we can compute the lower bound of the ergodic set for bequest as

$$\underline{b} = (1-\gamma-\psi)(1+r)\ \underline{b}$$

which completes the description of the support of the limiting distribution.

I postpone the proof showing that the limiting distributions for wealth and bequest under perfect enforcement exist and are stationary until I described the limiting distributions under imperfect enforcement. ■

Proof of Proposition 5. The result follows from a couple of theorems by Hadar and Russell (1971) who I refer the reader for proofs. Adapted to this application, they are.

THEOREM 1. Let $Y_i^A = \mu(1+r)zE_{F_i(N)}\left[e(N)^{\frac{1}{1-\alpha-\beta}}\right]$ for $i = 1, 2$. If $\frac{de(N)^{\frac{1}{1-\alpha-\beta}}}{dN} \geq 0\ \forall N$, being this inequality strict for some N, and if $F_2(N)$ rst order stochastically dominates $F_1(N)$, then $Y_2^A > Y_1^A$.

THEOREM 2. Let $Y_i^A = \mu(1+r)zE_{F_i(N)}\left[e(N)^{\frac{1}{1-\alpha-\beta}}\right]$ for $i = 1, 2$. If $\frac{d^2e(N)^{\frac{1}{1-\alpha-\beta}}}{dN^2} \leq 0\ \forall N$, being this inequality strict for some N, and if $F_2(N)$ second order stochastically dominates $F_1(N)$, then $Y_2^A > Y_1^A$. ■

Proof of Proposition 7. In this proof I show how to derive $v_t(b)$, since the rest follows directly from this result. Assume a given distribution of bequests $g_t(b)$. By Expression (??), next period bequest b' depends on previews bequest b only through N, implying that entrepreneurs with different inheritances might leave the same bequest to their successor as long as their end of youth wealth are equivalent and they are successful.. Unsuccessful entrepreneurs only leave bequest if and only their wealth was more than the amount invested in the project (if $N > N^*$). Also see that it is possible for a worker of the old generation to leave the same bequest than a successful entrepreneur as long as $B^E(N_j) = B^W(N_i)$ or than an unsuccessful one $\phi \max(N - N^*) = B^W(N_i)$. This means the mass of agents that inherit the same bequest can be offspring of either entrepreneurs (successful or not) or workers.

To compute the mass of agents rst note that at every point in time the mass of agents can be described by a density function $f_t(b,l)$ on their characteristics (b,l). This density function integrates one over the support of this distribution. With this at hand, we are able to compute the mass of successful entrepreneurs with the same wealth $N = b + (1+r)wl$, that bequest the same amount. Summing across them we obtain

$$\int_{N-(1+r)w\bar{l}}^{N} \mu e(N) f_t\left(x, l = \frac{N-x}{(1+r)w}\right) dx = \mu e(N) \int_{N-(1+r)w\bar{l}}^{N} h\left(\frac{N-x}{(1+r)w}\right) dG_t(x)$$

which is only a function of the entrepreneur wealth. Notice that this mass can be also expressed in terms of the amount this entrepreneurs will leave to their successors. The link between this variables is given by $B^E(N)$ or by $B^{E-1}(b)$ for all $b \geq \phi ae(0)^n$.

By the same reasoning all agents becoming workers might leave the same bequest than these entrepreneurs as long as they have a wealth $y = b + (1+r)wl$ such that

$$B^E(N) = \phi\left[(1+r)wl + b + w)\right]$$

where $\phi = (1-\gamma-\psi)(1+r)$. Also this works for all $B^E(N) \geq \phi w$

The mass of these agents is determined by

$$(1-\mu) \int_{\frac{B^E(N)-w\phi-\phi(1+r)w\bar{l}}{\phi}}^{\frac{B^E(N)-w\phi}{\phi}} h\left(\frac{B^E(N)-\phi(x+w)}{(1+r)w\phi}\right) dG_t(x)$$

Also, unsuccessful entrepreneurs might leave bequest if they had more than enough to fully nance their investment projects. They will also leave $B^E(N)$ as bequest as long as their wealth at youth is $B^E(N) = \phi\left[(1+r)wl + b - N^*\right]$. Their mass is

$$\mu[1-e^*] \int_{\frac{B^E(N)+\phi N^*-\phi(1+r)w\bar{l}}{\phi}}^{\frac{B^E(N)+\phi N^*}{\phi}} h\left(\frac{B^E(N)+N^*\phi-x\phi}{\phi(1+r)w}\right) dG_t(x)$$

where the effort level exerted is efficient because these entrepreneurs invested in the rm N^*.

Finally, by $v_t(b)$ can be computed by summing across all these agents, and taking into account the link between N and b

$$\begin{aligned} v_t(b) &= \mu e\left(B^{E-1}(b)\right) I(b \geq \phi a e(0)^n) \int_{B^{E-1}(b)-(1+r)w\bar{l}}^{B^{E-1}(b)} h\left(\frac{B^{E-1}(b)-x}{(1+r)w}\right) dG_t(x) + \\ &\quad \mu[1-e^*] \int_{\frac{b+\phi N^*-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b+\phi N^*}{\phi}} h\left(\frac{b+N^*\phi-x\phi}{\phi(1+r)w}\right) dG_t(x) + \\ &\quad (1-\mu) I(b \geq \phi w) \int_{\frac{b-w\phi-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b-w\phi}{\phi}} h\left(\frac{b-\phi(x+w)}{(1+r)w\phi}\right) dG_t(x) \end{aligned}$$

where $I(.)$ are indicator function. ■

Proof of Proposition 8. The upper and lower bounds of the ergodic set for bequest are easily derived. Moreover, for any (nite) upper bound for the initial distribution of bequest ($\bar{b}_0$), there is a sequence of upper bound for successive distributions given by $B^E\left(\bar{b}_{n-1} + (1+r)wl\right)$. To show that the distribution of bequest is stationary follows from proving that Expressions (44) and (45) de ne a mapping $v_{t+1}(b) = T(v_t(b))$ that is contracting.

By plugging (44) into (45) we obtain

$$\begin{aligned}
v_{t+1}(b) &= \mu e\left(B^{E-1}(b)\right) I(b \geq \phi ae(0)^n) \int_{B^{E-1}(b)-(1+r)w\bar{l}}^{B^{E-1}(b)} v_t(x) h\left(\frac{B^{E-1}(b)-x}{(1+r)w}\right) dx + \\
\mu e\left(B^{E-1}(b)\right) I(b &\geq \phi ae(0)^n) \left[1 - \int_{x\in(0,\bar{b}_t)} v_t(x)dx\right] \int_{B^{E-1}(b)-(1+r)w\bar{l}}^{B^{E-1}(b)} h\left(\frac{B^{E-1}(b)-x}{(1+r)w}\right) dx + \\
&\quad \mu[1-e^*] \int_{\frac{b+\phi N^*-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b+\phi N^*}{\phi}} v_t(x) h\left(\frac{b+N^*\phi - x\phi}{\phi(1+r)w}\right) dx + \\
&\quad \mu[1-e^*] \left[1 - \int_{x\in(0,\bar{b}_t)} v_t(x)dx\right] \int_{\frac{b+\phi N^*-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b+\phi N^*}{\phi}} h\left(\frac{b+N^*\phi - x\phi}{\phi(1+r)w}\right) dx + \\
&\quad (1-\mu) I(b \geq \phi w) \int_{\frac{b-w\phi-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b-w\phi}{\phi}} v_t(x) h\left(\frac{b-\phi(x+w)}{(1+r)w\phi}\right) dx + \\
(1-\mu) I(b &\geq \phi w) \left[1 - \int_{x\in(0,\bar{b}_t)} v_t(x)dx\right] \int_{\frac{b-w\phi-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b-w\phi}{\phi}} h\left(\frac{b-\phi(x+w)}{(1+r)w\phi}\right) dx
\end{aligned}$$

Also, after some algebra it can be show that

$$T\left(v_t(b)+a\right)=T\left(v_t(b)\right)+a[.]-a[.]\,\bar{b}_t$$

where [.] is given by

$$\begin{aligned}
[.] &= \mu e\left(B^{E-1}(b)\right)I(b\geq \phi ae(0)^n)\int\limits_{B^{E-1}(b)-(1+r)w\bar{l}}^{B^{E-1}(b)} h\left(\frac{B^{E-1}(b)-x}{(1+r)w}\right)dx+ \\
&\quad \mu[1-e^*]\int\limits_{\frac{b+\phi N^*-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b+\phi N^*}{\phi}} h\left(\frac{b+N^*\phi-x\phi}{\phi(1+r)w}\right)dx+ \\
(1-\mu)I(b &\geq \phi w)\int\limits_{\frac{b-w\phi-\phi(1+r)w\bar{l}}{\phi}}^{\frac{b-w\phi}{\phi}} h\left(\frac{b-\phi(x+w)}{(1+r)w\phi}\right)dx
\end{aligned}$$

where all the integrals in this expression are less than or equal to one. This implies that $[.]\leq 1$. For this reason it can be shown Blackwell s sufficient conditions for a contraction are satis ed for any nite $\bar{b}_0$.[23] Hence a unique stationary distribution $v(b)$ exists and is given by the xed point of the mapping $(v(b)=T(v(b))$. From this result, we conclude that a unique stationary distribution for bequest $(G_\infty(b))$ and wealth $(W_\infty(b))$ exist. Moreover, because the mapping T is contracting, $G_\infty(b)$ is the asymptotic distribution of the sequence of bequest distribution $\{G_0, G_{1,}...\}$ de ned by (44) and (45) and the initial G_0. ■

[23] See Pg. 54 of Stokey and Lucas.

Proof of Proposition 9. I split the proof in 3 steps. In step 1 I show that if G_t rst order stochastically dominates (FSD) G_{t-1}, then F_t FSD F_{t-1}. In step 2 I prove that if F_t FSD F_{t-1} then G_t FSD G_{t-1}. Finally in step 3 I show the result the Y_t^A converges monotonically toward $\bar{Y}^A$.

Step 1. $N = b + (1+r)wl_j$, $l_j \perp b$. Note that $F_t(N)$ can be written as

$$F_t(N) = \int_{-\infty}^{\bar{l}} \int_{-\infty}^{N-(1+r)wl} h(l)dG_t(b)dl$$

or

$$F_t(N) = \int_{-\infty}^{\bar{l}} h(l)\, G_t\left(N - (1+r)wl\right) dl$$

Thus,

$$F_t(N) - F_{t-1}(N) = \int_{-\infty}^{\bar{l}} h(l) \left[G_t\left(N - (1+r)wl\right) - G_{t-1}\left(N - (1+r)wl\right)\right] dl$$

Then, if G_t FSD G_{t-1} then F_t FSD F_{t-1}.

Step 2. See that

$$\begin{aligned} G_{t+1}(b) &= G_{t+1}(0) + \mu I(b > \phi a e(0)^n) \int_0^{B^{E-1}(b)} e(N)dF_t(N) + \\ & \mu(1-e^*) \int_{N^*}^{\frac{b+\phi N^*}{\phi}} dF_t(N) + \\ (1-\mu)I(b &> \phi w) \int_0^{B^{W-1}(b)} dF_t(N) \end{aligned}$$

where $G_{t+1}(0) = \mu \int_0^{N^*} [1 - e(N)]\, dF_t(N)$. Now also see that

$$G_{t+1}(0) + \mu(1 - e^*) \int_{N^*}^{\frac{b+\phi N^*}{\phi}} dF_t(N) = \mu \int_0^{\frac{b+\phi N^*}{\phi}} [1 - e(N)]\, dF_t(N)$$

Then

$$\begin{aligned} G_{t+1}(b) &= \mu \int_0^{\frac{b+\phi N^*}{\phi}} [1 - e(N)]\, dF_t(N) + \mu I(b > \phi a e(0)^n) \int_0^{B^{E-1}(b)} e(N) dF_t(N) + \\ (1-\mu) I(b &> \phi w) F_t(B^{W-1}(b)) \end{aligned} \tag{2.46}$$

Let the first term be R_1. Then this expression can be written as

$$R_1 = I(b > \phi a e(0)^n) R_1 + I(b \leq \phi a e(0)^n) R_1$$

Furthermore, it is important to note that $B^{E-1}(b) < \frac{b+\phi N^*}{\phi}$. With this, Equation 46 becomes

$$\begin{aligned} G_{t+1}(b) &= \mu I(b \leq \phi a e(0)^n) \int_0^{\frac{b+\phi N^*}{\phi}} [1 - e(N)]\, dF_t(N) + \\ \mu I(b &> \phi a e(0)^n) \int_0^{B^{E-1}(b)} dF_t(N) + \\ \mu I(b &> \phi a e(0)^n) \int_{B^{E-1}(b)}^{\frac{b+\phi N^*}{\phi}} [1 - e(N)]\, dF_t(N) + \\ (1-\mu) I(b &> \phi w) F_t(B^{W-1}(b)) \end{aligned} \tag{2.47}$$

Since

$$\int_{x}^{X} [1 - e(N)]\, dF_t(N) = F_t(X)\,[1 - e(X)] - F_t(x)\,[1 - e(x)] + \int_{x}^{X} F_t(N)e'(N)dN$$

then Expression (47), after some simpli cation over the second term becomes

$$\begin{aligned} G_{t+1}(b) &= \mu I(b \leq \phi ae(0)^n) \left\{ \begin{array}{c} F_t\left(\frac{b+\phi N^*}{\phi}\right)\left[1 - e\left(\frac{b+\phi N^*}{\phi}\right)\right] + \\ + \int_{0}^{\frac{b+\phi N^*}{\phi}} F_t(N)e'(N)dN \end{array} \right\} + \\ \mu I(b &> \phi ae(0)^n) \left\{ \begin{array}{c} F_t\left(\frac{b+\phi N^*}{\phi}\right)\left[1 - e\left(\frac{b+\phi N^*}{\phi}\right)\right] + \\ F_t\left(B^{E-1}(b)\right) e\left(B^{E-1}(b)\right) + \int_{B^{E-1}(b)}^{\frac{b+\phi N^*}{\phi}} F_t(N)e'(N)dN \end{array} \right\} + \\ (1-\mu)I(b &> \phi w)F_t(B^{W-1}(b)) \end{aligned} \tag{2.48}$$

Because $e'(N) > 0$, it is straight forward that if $F_t(N) \leq F_{t-1}(N)\ \forall N$ and strictly lower for some N then G_{t+1}FSD G_t.

Step 3. From steps 1 and 2 it should be clear that if for some period t, G_t FSD G_{t-1}, then $F_{t+\tau}(N)$ FSD $F_{t+\tau-1}(N)\ \forall\tau > 0$. Then by Theorem 1 or 2, we get that $Y^A_{t+\tau} > Y^A_{t+\tau-1}\forall\tau > 0$, which gives the monotonicity result. Finally, by Proposition 8, $\lim_{t\to\infty} F_t(N) = F_\infty(N)$ implies that $\lim_{t\to\infty} Y^A_t = \bar{Y}^A$. ■

Bibliography

[1] Antinol , G. and E. Huybens: On Domestic Financial Market Frictions, Unrestricted International Capital Flows, and Crises in Small Open Economies, Mimeo, November 1998.

[2] Bernanke, B. and M. Gertler: Financial Fragility and Economic Performance, QJE, February 1990.

[3] Bernanke, B. and M Gertler: Agency Costs, Net Worth, and Business Fluctuations, AER 79(1), March 1991.

[4] Bernanke, B. S. and M. Gertler: Inside the Black Box: The Credit Channel of Monetary Policy Transmission, Journal of Economic Perspectives 9(4), Fall 1995.

[5] Bernanke, B., M. Gertler and S. Gilchrist: The Financial Accelerator in a Quantitative Business Cycle Framework , NBER working paper 6455, 1998.

[6] Bhattacharya, Joydeep: Credit market imperfections, income distribution, and capital accumulation , Economic Theory 11, 171-200, 1998.

[7] Boyd, John H. and Bruce D. Smith: Capital Market Imperfections, International Credit Markets, and Nonconvergence , JET 73, 335-364, 1997.

[8] Cooley, T.F. and Vincenzo Quadrini: Financial Markets and Firm Dynamics , Mimeo 1998.

[9] Correia, I., J.C. Neves and S. Rebelo: Business Cycles in a Small Open Economy , European Economic Review 39, 1995.

[10] Cooley, T. F. and K. Nam: Asymmetric Information, Financial Intermediation, and Business Cycles, Economic Theory, Nov 1998.

[11] Davis, S. J., J. C. Haltiwanger and S. Schuh: Job Creation and Destruction. MIT Press, 1996.

[12] De Meza, D. and D. Webb: Risk, Adverse selection and Capital Market Failure, The Economic Journal 100, March 1990.

[13] Diamond, D. W.: Financial Intermediation and Delegated Monitoring, Rev. of Econ. Studies LI, 1984.

[14] Diamond, Douglas: Reputation Acquisition in Debt Markets, JPE 97(4), 1989.

[15] Evans, D.S.: Tests of Alternative Theories of Firm Growth, JPE 95(4), 1987.

[16] Fuerst, T.: Monetary and Financial Interactions in the Business Cycle, JMCB 27(4), November 1995.

[17] Galor, Oded and Joseph Zeira: Income Distribution and Macroeconomics , RES 60, 35-52, 1993.

[18] Gertler, Mark: Financial Capacity and Output Fluctuations in an Economy with Multi-Period Financial Relationships, Rev. of Econ. Studies 59, 1992.

[19] Gertler, M. and S, Gilchrist: Monetary Policy, Business Cycles, and the Behavior of small manufacturing Firms, QJE CIX, May 1994.

[20] Gertler, M.: Comment on Monetary and Financial Interactions in the Business Cycle, JMCB 27(4), November 1995.

[21] Greenwald, B. C. and J. E. Stiglitz: Financial Market Imperfections and Business Cycles, QJE, February 1993.

[22] Greenwood, Jeremy and Boyan Jovanovic: Financial Development, Growth, and the Distribution of Income , JPE 98, 1990.

[23] Hadar, J. and W. Rusell. Stochastic Dominance and Diversi cation , Journal of Economic Theory 3, 288-305, 1971.

[24] Holmstrom, B. and J. Tirole: Financial Intermediation, Loanable Funds, and the Real Sector, QJE CXII (3), August 1997.

[25] Jovanovic, Boyan: Selection and the Evolution of Industry, Econometrica 50 (3), May 1982.

[26] King, Robert and Ross Levine: Finance, entrepreneurship, and Growth , JME 32, 513-542, 1993.

[27] Kiyotaki, N. and J. Moore: Credit Cycles, JPE 105(2), 1997.

[28] Llowyd-Ellis, Huw and Dan Bernhardt: Enterprise, Inequality and Economic Development , RES 67, 147-168, 2000.

[29] Mendoza, E. G.: Real Business Cycles in a Small Open Economy, AER, September 1991.

[30] Modigliani F.F. and M.H. Miller: The cost of capital corporation nance, and the theory of investment , AER 48, 1958.

[31] Petersen, M. A. and Raghuram G. Rajan: The Bene ts of Lending Relationships: Evidence from the Small Business Data , The Journal of Finance, March 1994.

[32] Petersen, M. A. and Raghuram G. Rajan: The Effect of Credit Market Competition on Lending Relationships , QJE, May 1995.

[33] Piketty, Thomas: The Dynamics of the wealth Distribution and the Interest Rate with Credit Rationing , RES 64, 173-189.

[34] Rajan, Raghuram G. and Luigi Zingales. Financial Dependence and Growth , NBER Working Paper 5758, September 1996.

[35] Scheinkman, J. A. and L. Weiss: Borrowing constraints and Aggregate Economic Activity, Econometrica 54(1), January 1986.

[36] Stokey, Nancy and Robert Lucas. Recursive methods in Economic Dynamics, Harvard University Press, 1989.

[37] Torvik, Ragnar: Talent, Growth and Income Distribution , Scand. J. of Economics 95(4), 581-596, 1993.

[38] Williamson, S. D.: Costly Monitoring, Financial Intermediation, and equilibrium credit rationing, JME 18, 1986.

[39] Williamson, S. D.: Financial Intermediation, Business Failures, and Real Business Cycles, JPE 95(6), 1987.