C31- Game Theory

Spring 2004 - Problem set 1 (grades are in brackets)

1.[35] Consider the following Cournot duopoly. Firm 1 and 2 face the following inverse demand function

$$p = a - Q \equiv a - q_1 - q_2, \quad a > 0,$$

where q_i is the quantity produced by firm $i = 1, 2$. Both firms produce at the same constant marginal cost $c < a$. Firm 1 aims at maximizing her profits, whereas firm 2 maximimizes a (convex) combination of profits and revenues. More precisely, firm 2 maximizes

$$\beta R_2(q_1, q_2) + (1 - \beta)\Pi_2(q_1, q_2),$$

where R_2 and Π_2 represent firm 2's revenues and profits respectively, and $\beta \in (0, 1)$.

a. Prove or disprove that the amended Cournot duopoly coincides with the asymmetric Cournot duopoly in which firm 1's marginal cost is equal to c, whereas firm 2's marginal cost is equal to $(1 - \beta)c < c$.

b. Find the (unique) Nash equilibrium.

2.[35] Three people are engaged in a joint project. If each person i puts in the effort e_i, $e_i \in [0, 1/2]$, the outcome of the project is worth $6\sqrt{e_1 \cdot e_2 \cdot e_3}$. The cost of effort to person i is $c(e_i) = e_i$, $i \in \{1, 2, 3\}$. The worth of the project is split equally among the three persons, regardless of their effort levels.

a. Prove or disprove that $e_1^* = e_2^* = e_3^* = 0$ is a Nash equilibrium of this game.

b. Are there other *symmetric* Nash equilibria in the relevant range of effort levels? Explain.

3.[30] Consider a bargaining situation in which two individuals are considering undertaking a business venture that will earn them 100 pounds in profit, but they must agree on how to split the 100 pounds. Bargaining works as follows: The two individuals each make a demand simultaneously. If their demands sum to more than 100 pounds, then they fail to agree, and each gets nothing. If their demands sum to less than 100 pounds, they do the project, each gets his demand, and the rest goes to charity.

Prove or disprove that the strategy profiles (d_1, d_2) such that $d_i > 100$, $i = 1, 2$, constitute Nash equilibria of the bargaining game in which players use *weakly dominated* strategies.