THE ENFORCEMENT OF COLLUSION IN OLIGOPOLY

by

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ABSTRACT

Title: THE ENFORCEMENT OF COLLUSION IN OLIGOPOLY

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Submitted to the Department of Economics on May 1, 1981 in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics.

The three essays of this thesis examine oligopolistic outcomes from the perspective of rational disequilibrium adjustment.

The first essay studies the "local almost perfect" adjustment path of an industry with a fixed number of identical output controlling firms. Firms can respond to opponent's output deviations instantly but not costlessly. The long run industry equilibrium output is shown to lie between the Cournot-Nash and monopoly level depending on market conditions.

The second essay is closely related to the first - it asks what long run outcomes will be in an industry with an arbitrary technology but no costs of responding to opponents. Subject to technical qualifications I show that in the long run complete collusion occurs.

The third and final essay gives a rigorous analysis of "local almost perfect" equilibrium. In a dynamic game with adjustment costs firms do not have global information but are assumed to extrapolate linearly from complete local information. In an environment in which such extrapolation works well games theoretic regression is shown not to be a problem: the adjustment path is essentially unique.

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CHAPTER I: THE ENFORCEMENT OF COLLUSION IN A QUADRATIC SYMMETRIC OLIGOPOLY

0. Introduction

How and to what extent do self-interested oligopolistic firms successfully collude? The story of oligopolistic interaction ought to be a simple one. With the same firms in a static market over time uncooperative rivals can be punished and helpful rivals rewarded. Only to the extent that retaliatory strategies have associated frictional costs should less than fully collusive behavior be expected.

Current literature is divided on the question of whether oligopolists successfully collude. One class of models shows that various ad hoc assumptions about firm behavior lead to collusion.[1] Rigorously specified supergame models[2] lead to less satisfactory conclusions: almost any industry outcome at all is consistent with optimal play by firms with rational expectations.[3]

In a recent paper [10] I argued that when the bounded rationality of firms is taken account of there is only one economically significant adjustment process.[4] I argued that firms cannot be expected to know how their opponents will behave in every contingency, including contingencies that never occur. I argued that instead they will use local information near the status quo to extrapolate future income paths. I then showed that essentially only one adjustment path is consistent with firms extrapolating future income linearly. This I called the local almost perfect adjustment path.

This paper studies the long-run steady states of the local almost perfect adjustment path of a simple oligopoly. A fixed number of identical firms control their own output, produce at constant marginal cost and face linear demand.[5] Only market interactions are considered. Legal cartels and non-market interactions such as violence are ruled out. Firms are shown to follow a simple, sensible adjustment procedure. They choose to reward cooperative opponents and punish uncooperative ones. Because firms are assumed to adjust gradually these are credible threats. In this environment I attempt to relate long-run industry output to exogenously determined features of the market. My main conclusions are

- Output exceeds the monopoly level only if there are frictional costs of engaging in retaliatory policies.
- When enforcement costs are sufficiently high, output rises to the static Cournot-Nash level.
- Output rises with the number of firms and the discount rate and falls with market profitability.
- If firms exchange enough information to avoid counter-reacting to each other's retaliation, collusion is enhanced.

The paper has five sections. Section one describes a market in which firms can communicate threats. Section two derives the local almost perfect adjustment path for such a market. Section three describes the nature and stability of the long-run steady states of the adjustment path. Section four examines a duopoly in which firms cannot communicate, but do observe each other's output. Section five reexamines some widely believed myths about oligopoly in light of the findings of this paper.

1. The Model

This section describes a simple model of oligopoly without entry in which firms communicate threats, but cannot enter into legally binding contracts. The first half the section describes the actions available to firms and the information structure by which threats are communicated. The second half describes firm income. Discussion of firm behavior is deferred to section two.

The Environment: There are N identical firms, entry is prohibited and each firm j controls its own output x^j. Information is exchanged costlessly by a fixed information structure. At time t all firms j announce that they will respond to autonomous output changes by other firms k at a rate R^j_k.[6] At time $t+\Delta t$ all firms k announce their autonomous output changes of y^k.[7] At time $t+2\Delta t$ each firm j computes its total output change as

$$\Delta x^j = y^j + \sum_{k \neq j} R^j_k y^k = \sum_k R^j_k y^k \tag{1-1}$$

where $R^j_j \equiv 1$. Firms are assumed to observe one another's actual output. When Δt is infinitesimal relative to the distant rate j's opponents observe immediately whether or not he fulfilled his announced commitment given in (1-1). Thus lying is detected instantaneously and (1-1) may be taken to determine the amount by which j actually changes his output. In the continuous time limit as Δt goes to zero firm j chooses a vector of response rates (also called reaction coefficients) R^j and an autonomous rate of change of output y^j. Firm j's actual output follows the equation

$$\dot{x}^j = \sum_k R^j_k y^k \tag{1-2}$$

It is important that in this environment firms react only to opponents' autonomous output change. They do not counter-react to opponents' retaliation. The ability of firms to communicate is crucial: it is only the information generated by communication which enables firms to distinguish between voluntary and reactive movements by rival firms. The case in which firms cannot communicate is examined in section four.

In addition to changing output firm j may gradually alter its commitment R^j over time. This is given as

$$\dot{R}^j_k = S^j_k \tag{1-3}$$

where S^j_k is the rate at which firm j alters R^j_k. Thus firm j chooses paths for the control variables y^j and S^j in an effort to control the state variables x and R.

<u>Firm Income</u>: The income of firm j is a function of the state variables

$$a^j(x,R) = \pi^j(x) - bC^j(R^j) \tag{1-4}$$

where π^j is sales revenue minus production costs, $bC^j(R^j)$ are frictional reaction costs discussed below and $b>0$ is a small scalar constant. Adjustment costs--costs which depend on j's control variables--could also be included. For notational simplicity I prefer a model of partial adjustment as described in section two.

All of the identical firms produce at constant marginal cost and face linear demand[8] so that

$$\pi^j(x) = E(1-q)x^j \tag{1-5}$$

where E is price minus marginal cost at zero industry output, q is industry output as a fraction of the competitive level

$$q \equiv (\beta/E) \sum_k x^k \tag{1-6}$$

and β is the slope of the demand curve.

What are the frictional costs bC^j? This paper focuses on the long-run industry steady state and describes the adjustment towards this steady state. The adjustment is described as a smooth path capturing long-run trends--short-run output fluctuations around this trend do not appear. In reality there _will_ be short-run output fluctuations as firms engage in experimentation to learn about the environment they face, and because of various short-run random shocks. It is these short-run frictions (which are not explicitly modelled) which give rise to reaction costs. When firm j is committed to a policy of responding to k at a rate R_k^j he must hold costly inventories to meet short-term output movements by k. The larger $|R_k^j|$ the greater the inventories he must hold. For simplicity the functional form

$$bC^j(R^j) = bm \sum_{k \neq j} |R_k^j| \tag{1-7}$$

is assumed where $m > 0$ is a scalar constant.

Besides inventories there are other short-run frictions which make response costly--most notably it may be costly to observe opponents messages or output. Explicit models along these lines are in Green [4], Stigler [20] and especially Spence [19]. Since I am primarily interested in the implications rather than the sources of frictional costs the function (1-7) is given exogenously rather than derived from underlying economic data. Note that since b is small frictional costs are presumed small. This is no limitation on the generality of the results here since large costs are a special limiting case of small costs.

To summarize, firm j controls y^j and S^j. The state variables are x and R which move according to (1-2) and (1-3). Firm income is given by (1-4) and firms' objectives are the present value of future income computed using a common discount rate $0 < \rho < \infty$. The next section describes firm behavior.

2. Firm Behavior

This section describes firm behavior by applying the notion of a local almost perfect equilibrium developed in Levine [10]. This attempts to model the bounded rationality of firms by assuming that they compute present values of income streams by extrapolating existing rates of income growth linearly into the future. The first half of the section derives the local almost perfect adjustment paths of firms. The second half explicitly computes the equations of motion of the state variables when initial conditions are symmetric. The qualitative features of the adjustment equations are developed in section three.

Local Almost Perfect Adjustment: A strategy (or closed-loop strategy) for firm j is a function

$$(y^j, s^j) = F^j(x,R) = (f^j(x,R), g^j(x,R)) \qquad (2\text{-}1)$$

which assigns a vector of control variables to the vector of state variables. This already embodies a degree of bounded rationality since firm j's choice depends only on the current state variables and not the entire past history of the market. Suppose that all firms k play $\tilde{F}^k$. Then firm j receives

$$\tilde{A}^j(x,r) = \int_0^\infty a^j(x(t),R(t)) \exp(-\rho t)dt \tag{2-2}$$

where x(t) and R(t) satisfy the system of differential equations

$$\left.\begin{aligned} \dot{x}^j &= \textstyle\sum_k R_k^j \tilde{f}^k(x,R) \\ \dot{R}^j &= \tilde{g}^j(x,R) \end{aligned}\right\} \quad j = 1,\dots,N$$

$$x(0) = x \qquad R(0) = R \tag{2-3}$$

which are derived by substituting (2-1) into the equations of motion for the state variables (1-2) and (1-3).

If firm j has rational expectations it knows how $\tilde{A}^j$ depends on the state variables and simply chooses the controls to maximize this present value. It is unreasonable to suppose that firm j can in fact compute $\tilde{A}^j$ for this requires it to know how the industry will behave in all contingencies. I shall suppose instead that at (x,R) firm j knows an estimate $\hat{A}^j(x,R)$ formed by extrapolating local information in a manner indicated below. Firm j also knows the derivatives $D\hat{A}^j(x,R)$ (of $\hat{A}^j$ with respect to the state variables) which are assumed to approximate $D\tilde{A}^j(x,R)$. Firm j then chooses its control variables so as to move the state variables in a direction which (approximately) increases the (rationally expected) present value

$$\tilde{f}^j = b\,k \left\{ \sum_k \frac{\partial \hat{A}^j}{\partial x^k} \frac{\partial \dot{x}^k}{\partial y^j} + \sum_k \sum_\ell \frac{\partial \hat{A}^j}{\partial R^k_\ell} \frac{\partial \dot{R}^k_\ell}{\partial y^j} \right\}$$

$$= b\,k \left\{ \sum_k \frac{\partial \hat{A}^j}{\partial x^k} R^k_j \right\}$$

$$\tilde{g}^j_k = b \left\{ \sum_k \frac{\partial \hat{A}^j}{\partial x^k} \frac{\partial \dot{x}^k}{\partial S^j_k} + \sum_k \sum_\ell \frac{\partial \hat{A}^j}{\partial R^k_\ell} \frac{\partial \dot{R}^k_\ell}{\partial S^j_k} \right\}$$

$$= b \frac{\partial \hat{A}^j}{\partial R^j_k} \qquad (2\text{-}4)$$

where the equations of motion (1-2) and (1-3) are used to compute $\partial \dot{R}^k_\ell / \partial y^j = 0$, $\partial \dot{x}^k / \partial S^j_k = 0$ and $\partial \dot{R}^k_\ell / \partial S^j_k = 0 \quad \ell \neq j$; $k > 0$ is a constant adjustment coefficient, and $b > 0$ is the same small scalar constant that appears in (1-4), the import of which is described below.

What (2-4) describes is a partial adjustment model with $b\,k$ and b exogenous adjustment rates for the two types of controls. In this model the controls are set to increase the (approximate) level of present value over time. The factor b shows that the controls are small--equivalently that the state variables are adjusted gradually.

Implicitly it is expensive to choose large values of the controls. Indeed, in another paper [10], I show that if there are quadratic costs of choosing the controls the adjustment process (2-4) is almost optimal provided that $D\hat{A}^j$ is uniformly close to $D\tilde{A}^j$ the true present value derivative.[9] Naturally adjustment costs and partial adjustment are dual.

One important reason for a partial adjustment model is bounded rationality. If firm j wishes to globally and instantly set the optimal output level (for example) he must know his demand curve everywhere. If he is content to restrict himself to change output slowly, he need only learn a small segment of his demand curve each day to make an optimal choice subject to his self-imposed constraint. The point is that the faster a firm moves the more quickly it must learn. Bounded rationality and large costs of gathering information imply it will move rather slowly.

How do firms use local information to approximate the present value of income? If the future income stream is not too badly non-linear, simply extrapolating income linearly should be a good approximation--indeed, casual empiricism indicates this is a widely used technique. The remainder of this sub-section gives a heuristic derivation of the (approximately) unique adjustment process which is smooth enough to allow reasonably accurate linear extrapolation by firms. Formal statements and proofs of these results can be found in Levine [10].

The linearly extrapolated present value of income is

$$\overline{A}^j \equiv \int_0^\infty [a^j + \dot{a}^j t]\exp[-\rho t]dt$$

$$= \delta a^j + \delta^2 \dot{a}^j \qquad (2\text{-}5)$$

where $\delta=1/\rho$ is the discount factor. Expanding $a^j(t)$ in a Taylor series under the integral in (2-2) shows that the error in this approximation is no worse than $\delta^3 \sup|\ddot{a}^j|$. Similarly $D\bar{A}^j$ is within $\delta^3 \sup|D\ddot{a}^j|$ of $D\tilde{A}^j$. If all firms k play $\tilde{F}^k$ with $|\tilde{F}^k|, |D^p\tilde{F}^k| \leq b\gamma$ for sufficiently high order p it can be shown that $|\ddot{a}^j|, |D\ddot{a}^j| \leq H(m,E,\beta,\gamma)b^2$ where H is a fixed function. Thus, if the adjustment of all firms is slow enough, linear extrapolation will work well.

Suppose that b is small enough that firms are willing to accept an approximation error of order b^2. I will now derive estimates $\hat{A}^j$ for each firm which: (1) depend solely on local information extrapolated linearly; (2) given that all firms k set $\tilde{F}^k$ according to (2-4) the estimates $\hat{A}^j$ are within order b^2 of the actual present value $\tilde{A}^j$ and this is true also for $D\hat{A}^j$ and $D\tilde{A}^j$; and (3) the functions $\hat{A}^j$ are given in terms of the exogenous data describing the economic environment. I will not show here, although it is shown in Levine [10], that any other adjustment process in which firms make errors of no worse than order b^2 and which permit the possibility of linear extrapolation are approximately the same as the particular adjustment process derived below.

To find what the functions $\hat{A}^j$ must be, assume that they exist and that $\tilde{F}^j$ are the corresponding strategies from (2-4). Once the functions are computed it is then straightforward to verify that they have the desired properties. From (2-5) the linearly extrapolated present value is[10]

$$\bar{A}^j = \delta a^j + \delta^2 \dot{a}^j$$

$$= \delta a^j + \delta^2 \left\{ \sum_\ell \frac{\partial a^j}{\partial x^\ell} \left(\sum_k R^\ell_k \tilde{f}^k\right) + \sum_\ell \frac{\partial a^j}{\partial R^j_\ell} \tilde{g}^k_\ell \right\}$$

$$= \delta a^j + \delta^2 \left\{ \sum_\ell \pi^j_\ell \left(\sum_k R^\ell_k \tilde{f}^k\right) - bm \sum_\ell \mathrm{sgn}(R^j_\ell) \tilde{g}^k_\ell \right\} \qquad (2\text{-}6)$$

where $\pi^j_\ell \equiv \partial \pi^j / \partial x^\ell$,

$$\mathrm{sgn}(z) = \begin{cases} +1 & z > 0 \\ 0 & z = 0 \\ -1 & z < 0 \end{cases} \qquad (2\text{-}7)$$

and $\dot{a}^j$ is computed using $\dot{x}$ and $\dot{R}$ from (2-3) and (1-4). By assumption, this approximation is good to order b^2. Substitution of (2-4) into (2-6) shows

$$\bar{A}^j = \delta a^j + b\delta^2 \mathcal{R} \sum_\ell \pi^j_\ell \sum_k R^\ell_k \sum_m R^m_k \frac{\partial \hat{A}^k}{\partial x^m} + O(b^2)$$

$$= \delta a^j + b\delta^2 \mathcal{R} \sum_k \sum_m \left(\sum_\ell \pi^j_\ell R^\ell_k\right) R^m_k \frac{\partial \hat{A}^k}{\partial x^m} + O(b^2) \qquad (2\text{-}8)$$

where $O(b^2)$ denotes an error of order b^2. Thus

$$\frac{\partial \bar{A}^k}{\partial x^m} = \delta \pi^k_m + O(b) \qquad (2\text{-}9)$$

Recall that $D\hat{A}^k$ is assumed to differ from $D\tilde{A}^k$ by at most $O(b^2)$. Since $D\bar{A}^k$ also differs from $D\tilde{A}^k$ by at most $O(b^2)$, $D\hat{A}^k$ and $D\bar{A}^k$ differ by at most $O(b^2)$. Certainly, then

$$\frac{\partial \hat{A}^k}{\partial x^m} = \delta\pi_m^k + O(b) \qquad (2\text{-}10)$$

Substituting (2-10) into (2-9) shows

$$\bar{A}^j = \delta a^j + b\delta^3 \mathcal{R} \sum_k \sum_m \left(\sum_\ell \pi_\ell^j R_k^\ell\right) R_k^m \pi_m^k + O(b^2) \qquad (2\text{-}11)$$

Thus, if $\hat{A}^j$ is <u>defined</u> by

$$\hat{A}^j \equiv \delta a^j + b\delta^3 \mathcal{R} \sum_k \sum_m \left(\sum_\ell \pi_\ell^j R_k^\ell\right) R_k^m \pi_m^k \qquad (2\text{-}12)$$

it differs from $\bar{A}^j$ and thus $\tilde{A}^j$ by at most $O(b^2)$ and can be shown to have the properties assumed in its computation.

It is worth reflecting on how (2-12) solves the infinite regress problem that plagues game theory. It says that if firms are willing to extrapolate linearly they might as well assume in addition that in the future all firms will behave myopically--that (2-10) will hold without the error term. The point is that while each firm's behavior depends on what it thinks its opponents thinks it thinks, etc. in a model passing through time, the higher order terms in this chain of double-think have consequences which are in the distant future. While the optimal path for j depends on what k thinks j thinks as represented by the error term in (2-10) this has such a small impact on k's behavior, j may as well ignore it entirely. Thus, the infinite regress is replaced with a finite regress and at the final stage opponents assume everyone is myopic and a unique solution is determined.

My view is that when firms enter an economic environment they do so with the working hypothesis that they can extrapolate linearly following (2-12) without substantial loss. In the environment of this paper, when all firms behave this way, this conjecture is true and firms will never have reason to reject it. In this environment the only economically meaningful adjustment process is that given above.

Nature of the Solution: From (2-4) the partial adjustment equation and (2-12) giving the approximate present value

$$\dot{x}^j = \sum_k R^j_k \tilde{f}^k$$

$$= \sum_k R^j_k \left(b \mathfrak{k} \sum_\ell \frac{\partial \hat{A}^k}{\partial x^\ell} \frac{\partial \dot{x}^\ell}{\partial y^k}\right)$$

$$= b \mathfrak{k} \sum_k R^j_k \sum_\ell R^\ell_k \delta \pi^k_\ell + O(b^2)$$

$$= b \delta \mathfrak{k} \sum_k \sum_\ell R^j_k R^\ell_k \pi^k_\ell + O(b^2) \tag{2-13}$$

Since the first term in (2-13) is O(b) while the second term is $O(b^2)$ to a good approximation[11]

$$\dot{x}^j = b\delta\mathfrak{k}\sum_k\sum_\ell R^j_k R^\ell_k \pi^k_\ell \tag{2-14}$$

I am <u>not</u> arguing that firms will ignore the second term--to achieve the same degree of extrapolative accuracy as provided by linear extrapolation; they cannot. I am pointing out instead that the qualitative features of the dynamical system describing the evolution of x and R over time can be understood without reference to the second term in (2-13). In economic terms, I don't care whether a firm's market share is 10% or 12% even though the firm itself may care tremendously. In mathematical terms, including the second term when b is small enough merely perturbs the location of steady states slightly without affecting their stability.[12]

The qualitative analysis in this paper focuses on the case when initial conditions are symmetric.[13] Since the adjustment equations are symmetric the system will remain symmetric for all time. Thus the system is entirely described by just two variables which may be taken to be q, industry output as a fraction of the competitive level given in (1-6), and $r \equiv R^j_k$ $j \neq k$ the common reaction of any firm to any other firm. In this case equation (2-14) simplifies to

$$\dot{q} = (b\delta\mathfrak{k}\beta N/E)[1+(N-1)r][\pi^j+(N-1)r\pi^j_k] \tag{2-15}$$

where by symmetry and (1-5) describing the quadratic profit function

$$\pi_j^j = (E/N)[N-(1+N)q]$$

$$\pi_k^j = -(E/N)q \qquad j \neq k \qquad (2\text{-}16)$$

Turning to $\dot{R}_k^j$ from (2-4) the partial adjustment euqation and (2-12) giving the approximate present value

$$\dot{R}_k^j = \tilde{g}_k^j = b \frac{\partial \hat{A}^j}{\partial R_k^j}$$

$$= b\{-\delta bm \operatorname{sgn}(R_k^j)$$

$$+ b\delta^3 \mathcal{R} [\textstyle\sum_m R_k^m \pi_j^j \pi_m^k + \pi_j^k \sum_\ell \pi_\ell^j R_k^\ell]\}$$

$$= b^2\delta\{\delta^2 \mathcal{R} [\pi_j^j \textstyle\sum_m R_k^m \pi_m^k + \pi_j^k \sum_m R_k^m \pi_m^j] - m \operatorname{sgn}(R_k^j)\} \qquad (2\text{-}17)$$

Notice that since all terms in (2-17) are $O(b^2)$ none can be ignored even as an approximation. This is because frictional reaction costs are assumed small of the same order as the adjustment rate. When these costs are not small (2-17) is dominated by the term $-b\delta m \operatorname{sgn}(R_k^j)$ which is special case of (2-17)--the case m large. When frictional reaction costs dominate (2-17) simply says that reaction coefficients are set to minimize the cost of reacting. When reaction costs are small the second term in (2-17) is important. It shows how a firm should optimally punish and reward opponents--how to change R_k^j so as to get opponents to alter output in a preferred direction. The crucial insight (ignoring frictional response costs) is that the <u>only</u> effect reaction coefficients have is on the future behavior of opponents. As long as firms care about the future at least enough to use linear extrapolation rules it is precisely the effect of reaction coefficients on opponents' output that firms pay attention to in determining how to adjust these coefficients. This should be contrasted to (2-13) where the effect of autonomous output on opponents' future choice is relatively less important than the direct impact from having opponents respond instantaneously according to precommitted coefficients R_k^j.

When initial conditions are symmetric, (2-17) simplifies to

$$\dot{r} = b^2\delta\{\delta^2\mathcal{R}[\pi_j^j(\pi_j^j+(N-1)r\pi_k^j + \pi_k^j(r\pi_j^j+(1+(N-2)r)\pi_k^j)] - m\,\mathrm{sgn}(r)\} \qquad (2\text{-}18)$$

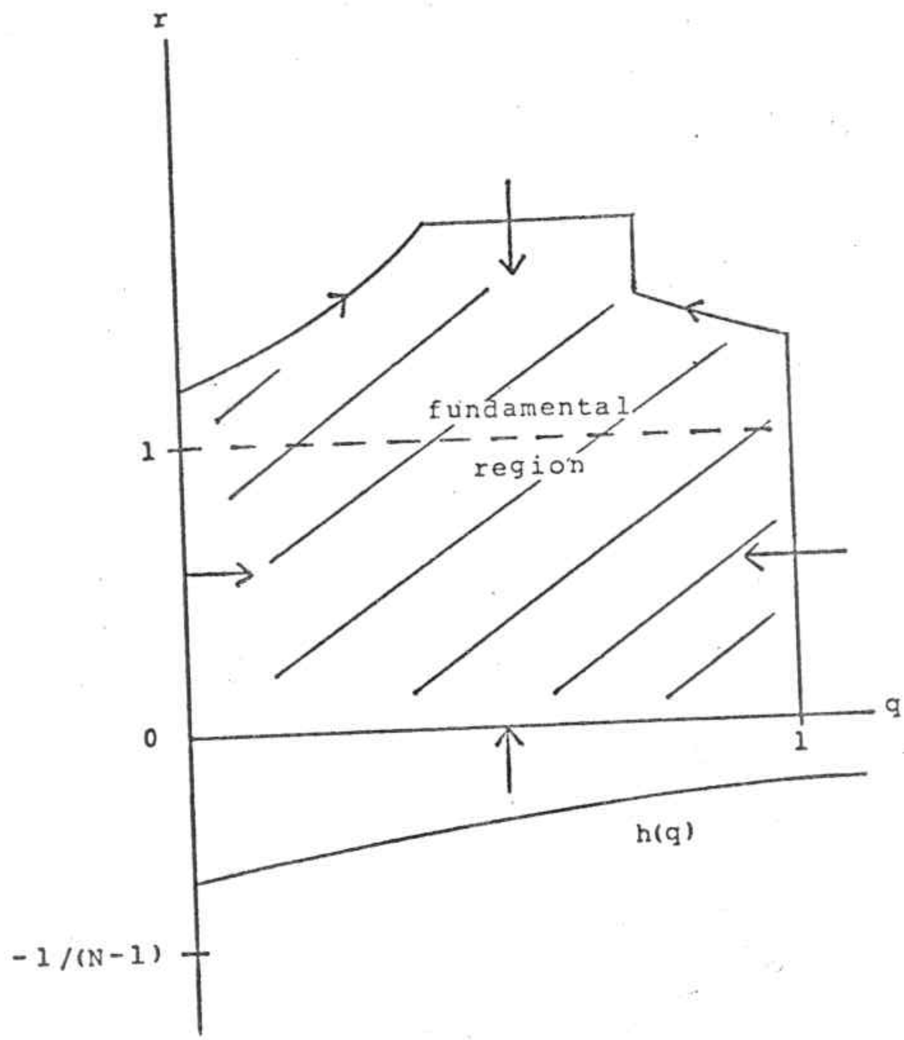
3. Oligopolistic Outcomes

This section studies the equations of motion derived in the previous section. It has three parts. The first part considers some global aspects of the dynamics. It shows that all stable steady states are contained in a compact positive invariant region which is reached (for all initial conditions with r not too greatly negative) in finite time. Part two considers steady state behavior in the short run. In the short run the response rate r is determined by initial conditions while industry output is at the "conjectural variational equilibrium" where firms conjecture that opponents will respond with the variation r. Part three analyzes the long run. In the long run r is determined endogenously: it is this feature which distinguishes this theory from previous oligopoly theories. In the long run, steady state output lies between the monopoly and Cournot-Nash output depending on the exogenous parameters of the market. The most significant result is that when there are no frictional costs of response output is at the monopoly level independent of other market parameters. Several comparative static exercises show how long-run output varies with market parameters when frictional costs are positive.

Global Aspects: Figure (3-1) sketches the fundamental region and the continuous strictly negative function h(q) in q-r space. Note that the fundamental region contains the region $0 \leq r \leq 1$ and $0 \leq q \leq 1$. Appendix (A) derives the properties of this region. They are: (1) it is positive invariant--if the system starts in this region it never leaves it, and (2) if the system starts in the region $r \geq h(q)$ it reaches the fundamental region in finite time.

Intuitively what this analysis indicates is that we should expect to observe $r \geq 0$ but not too large and $0 \leq q \leq 1$. The latter property is sensible: since q is industry output as a fraction of the competitive output, and since frictional costs are a negligible component of total costs $q \leq 1$ is (approximately) the region of non-negative profits. The region $r < 0$ does not make economic sense: when r is negative each firm rewards its opponents for hurting it and punishes its opponents for cooperating. Finally if r is extremely large each firm gives its opponents such large rewards for cutting their output that industry output will actually be below the monopoly level--not a desirable situation from anyone's point of view.

This analysis has one drawback. If the system starts with $r < h(q)$ the fundamental region may never be reached. In fact there can be steady states with $r < 0$. Evidently the myopic behavior of firms can cause the system to get stuck in a region which is, in the long run, undesirable. It is shown in Appendix (B), however, that any steady



Figure(3-1): The Fundamental Region

state with $r < 0$ is unstable. I conjecture also that the set of initial conditions with $r < h(q)$ for which the system doesn't reach the fundamental region has measure zero so that small random perturbations would eventually cause the fundamental region to be reached.

The fact that the system could in principle remain forever in a region where firms make negative profits does point up a weakness in the preceding formulation. In most cases it is reasonable to suppose that firms know only the local effect of small changes near the status quo. The important exception is that firms know they can make zero profits by closing down. This can be integrated into the analysis here by simply assuming firms shut down whenever $\hat{A}^j$ the extrapolated present value becomes negative.

In the remainder of the section attention is restricted to the fundamental region.

<u>The Short Run</u>: Inspection of the equations of motion (2-15) and (2-18) shows that q adjusts at a speed proportional to b while $\dot{r}$ is proportional to b^2. Since b is assumed small this means q adjusts much more quickly than r. In the short run q moves rapidly towards the curve $\dot{q}=0$, while r is close to its starting value. Examination of (2-15) shows that when $r > -1/(N-1)$ $\dot{q}=0$ implies

$$[\pi_j^j + (N-1)r\pi_k^j] = 0 \tag{3-1}$$

This is exactly the first order condition for choosing the optimal output level subject to the conjecture that opponents of j respond to output changes Δx^j by $r\Delta x^j$. Thus, in the short run, the steady state resembles the traditional conjectural variational equilibrium, as described for example by Seade [17].

From (3-1) and (2-16) giving the profit derivatives the curve $\dot{q}=0$ is seen to be

$$r = \frac{N - (1+N)q}{(N-1)q} \tag{3-2}$$

which is sketched in figure (3-2). The curve strictly decreases. This reflects the fact that the more j's opponents reward him for cutting output by responding with output cuts of their own, as reflected in large r, the lower j will choose to set his output. Note that (2-15) is only an approximation to (2-14)--incorporating the extra term of order b^2 would have the effect of shifting the curve (3-2) by an amount proportional to b.

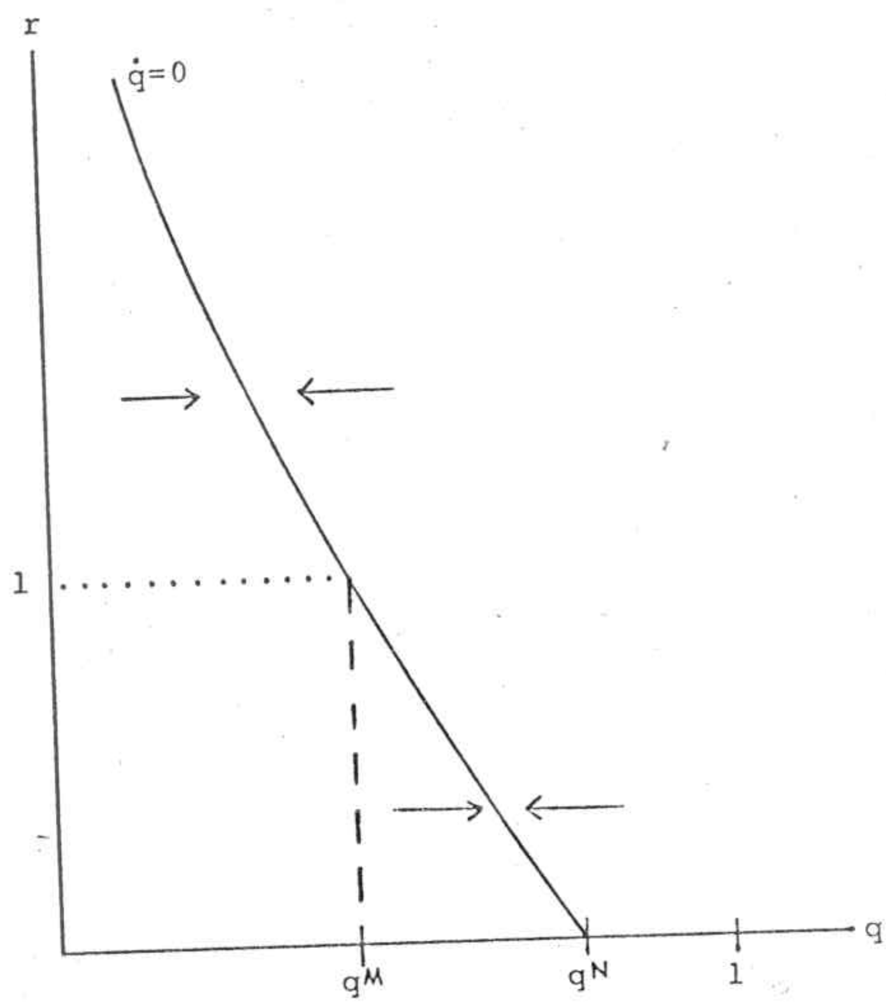
At the monopoly output

$$q^M \equiv (1/2) \tag{3-3}$$

the reaction r=1 while at the Cournot-Nash output

$$q^N \equiv \frac{N}{1 + N} \tag{3-4}$$

the reaction r=0. Finally, from (2-15) and (2-16)



Figure(3-3): The Short Run Steady State Curve

$$\frac{\partial\dot{q}}{\partial q} = -(b\delta k\beta)(1+(N-1)r)[(1+(N-1)r) + N] \tag{3-5}$$

which is negative for $r > -1/(N-1)$. Thus in the short run q moves towards the curve $\dot{q}=0$, and the short-run steady states are globally stable.

<u>The Long Run</u>: The theory of oligopoly developed in the previous sections differs from orthodox theory because in the long run r is determined endogenously. In addition to satisfying $\dot{q}=0$, in the long run steady state $\dot{r}=0$ must be satisfied. Using the equation of motion for r (2-18) and the profit derivatives (2-16) shows that $\dot{r}=0$ along the curve

$$r = \frac{(mN^2/E^2\delta^2 k) - [(N^2+2N+2)q^2 - 2N(1+N)q + N^2]}{[(N^2+2N-2)q - N^2]q} \tag{3-6}$$

Equating (3-6) with (3-2) and solving gives long-run steady state output as

$$q^S = \frac{3}{4} - \sqrt{\frac{1}{16} - M} \tag{3-7}$$

$$q^U = \frac{3}{4} + \sqrt{\frac{1}{16} - M} \tag{3-8}$$

where the constant

$$M \equiv \frac{(N-1)m}{2E^2\delta^2 \ell} > 0 \qquad (3-9)$$

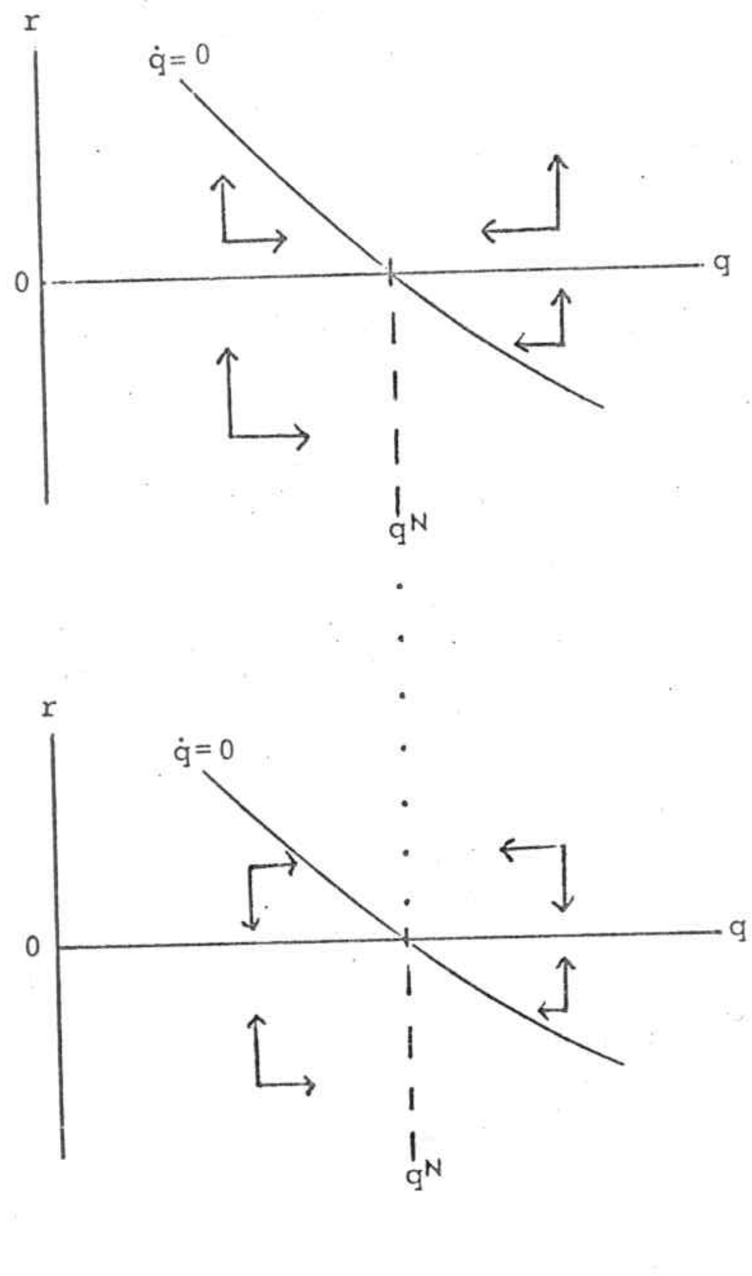
Since $r \geq 0$ is assumed, by (3-2) these steady states exist only when the corresponding output is no greater than q^N. In addition neither steady state exists if $M > (1/16)$. Appendix (C) shows that when they exist the steady state at q^S is stable and the steady state at q^U unstable.

There is one other possible steady state. There can be a steady state along $r=0$ since $\dot{r}=0$ is discontinuous there. Inspection of figure (3-2) shows that if this occurs it must be at the Cournot-Nash output q^N. As shown in figure (3-3) there will be a steady state at $r=0$, $q=q^N$ if and only if when evaluated at $q=q^N$ and $r > 0$ but small $\dot{r} < 0$. If this steady state exists, it is stable as figure (3-3) shows. A computation using (2-18) and (2-16) shows that for

$$M^N \equiv \frac{N-1}{2(1+N)^2} \qquad (3-10)$$

the condition for a steady state at q^N is $M > M^N$.

The next stage of analysis is to determine how the location of steady states depends on M, N fixed. After showing industry output as a fraction of the competitive level increases in M the issue of how M (and therefore market competitiveness) depends on market parameters is addressed. There are two cases in analyzing steady state



Figure(3-3): The Flow near q^N

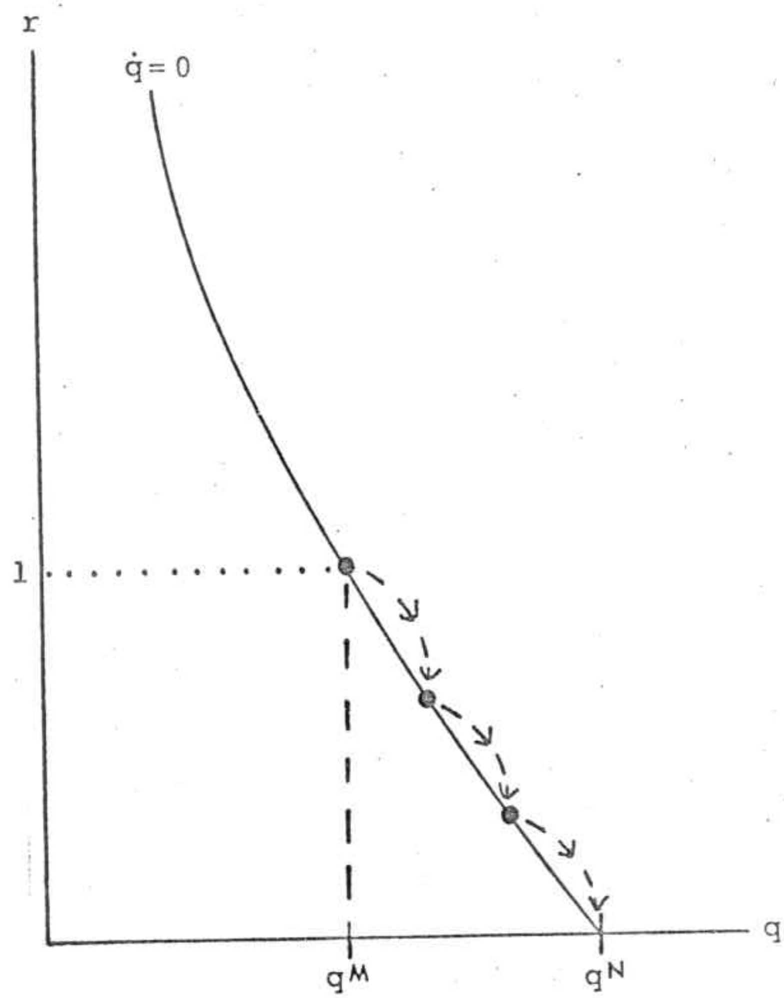
output N=2,3 and $N \geq 4$. The economic significance of the cutoff value N=4 is doubtful--it is probably an artifact of the choice of functional form.

Examination of (3-8) shows that when N=2,3 $q^U \geq q^N$ and thus there is no steady state at q^U. Examination of (3-7) shows q^S increases in M. When M=0, $q^S=q^M$. As M increases q^S increases until it reaches q^N when $M=M^N$. Thus when N=2,3 there is a unique steady state with $r \geq 0$, and it is stable. For $0 \leq M < M^N$ the steady state output is given by q^S. For $M \geq M^N$ steady state output is at the Cournot-Nash level. This situation is illustrated in figure (3-4).

When $N \geq 4$ the situation is more complex. Define the cutoff point

$$M^B = (1/16) \qquad (3\text{-}11)$$

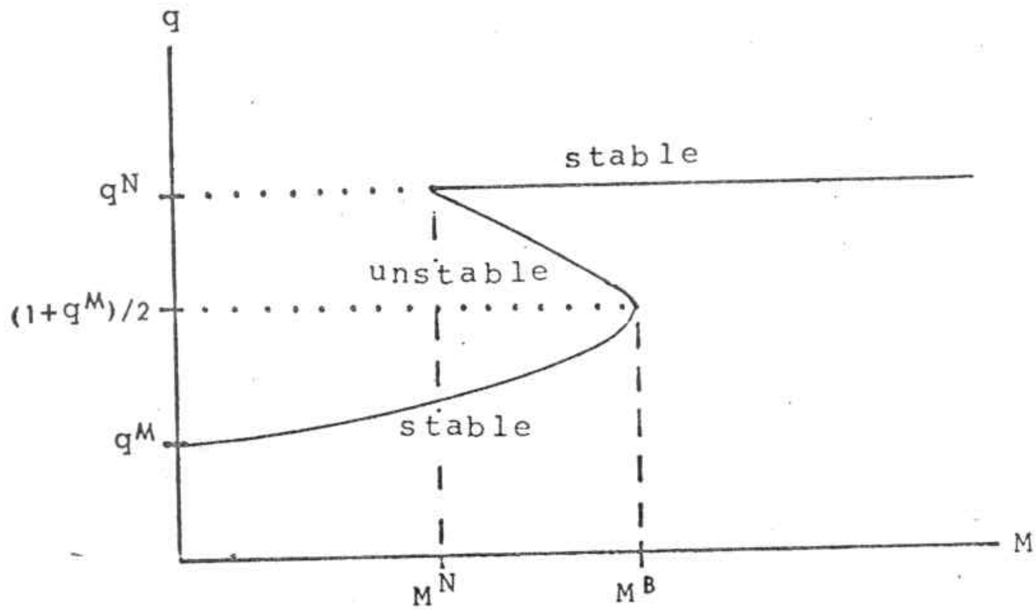
When M=0 there is a unique steady state (which is stable) at $q^S=q^M$ and r=1. As M increases to M^N there is still a unique steady state (stable) with output q^S. For $M^N < M < M^B$ there are three steady states with output q^S (stable), q^U (unstable) and q^N (stable). When $M > M^B$, q^S and q^U meet at $(3/4)=(q^M+1)/2$ and vanish leaving just one steady state (stable) at q^N. Figure (3-4) diagrams steady state output. As in the case N=2,3 output increases in M. However a discontinuity in steady state output can occur when $M=M^N$ or $M=M^B$ as the system jumps from one steady



Figure(3-4): The Case N=3,4
arrows denote movement in q^S for N fixed as M increases

state to another. In the language of catastrophe theory this is the double fold catastrophe.[12]

From (3-4) M and thus industry output as a fraction of the competitive level increases in N and m and decreases in E, δ and k. That increases in N lead to more competition shouldn't be too surprising. In this model m represents the marginal cost of enforcing a collusive arrangement--of increasing R_k^j. Thus when m is large there is less collusion. There is also a public goods problem in allocating enforcement costs among firms. When N is large each firm has less incentive to bear its share of the burden and long-run output is greater. The fact that competitiveness declines in E is perhaps a bit surprising. This happens for two reasons. First, m/E, which is the cost of enforcing collusion divided by a variable describing the marginal profitability of increasing output, declines. Thus E serves to scale m: it isn't enforcement costs, but the ratio of marginal enforcement costs to marginal market profitability that matters. Second, from the equation of motion for q (2-15), when E increases, firms adjust output more quickly due to the increased marginal profits from doing so. Increasing k also increases the speed with which firms adjust. Why does more rapid adjustment of output enhance collusion? The benefit of setting high values of R_k^j and thus realizing relatively collusive arrangements lies in the effect this has on opponents' <u>future</u> output. The more quickly



Figure(3-5): Steady State Industry Output

$N \geq 4$ and held fixed

they adjust, the more quickly these benefits are realized and with discounting fixed, the more valuable they are. Naturally, increasing the discount factor δ has the same effect, placing more weight on the future benefits of collusion, less on the immediate costs. Thus raising E, k or δ all have the effect of reducing competitiveness.

4. Duopoly without Communication

One drawback of the preceding analysis is that it requires that firms distinguish between voluntary output changes by rivals and reactions by rivals to voluntary output changes. Without communication between firms this distinction is impossible. This section analyzes a duopoly in which the firms observe each other's output, but cannot communicate. It has four parts. Part one describes an environment in which each firm reacts after an infinitesimal lag to its rival's observed output change. Part two derives the local almost perfect adjustment path for firms. Part three computes steady state output. Part four compares this output with the steady state output in the model of section three.

The Environment: Consider first an infinitesimal reaction lag, Δt. Firms react to opponents' output change in the previous period

$$\Delta x^j(t+\Delta t) = y^j + \bar{R}^j_k \Delta x^k(t) \qquad j \neq k \tag{4-1}$$

where y^j is firm j's voluntary output change and $\bar{R}^j_k$ is its (structural) reaction coefficient. Define

$$\bar{R} \equiv \begin{bmatrix} 0 & \bar{R}_2^1 \\ \bar{R}_1^2 & 0 \end{bmatrix} \tag{4-2}$$

Let

$$\lambda(\bar{R}) \equiv \sqrt{|\bar{R}_2^1 \bar{R}_1^2|} \tag{4-3}$$

be the largest absolute eigenvalue of $\bar{R}$. Then for $\lambda < 1$ the discrete time dynamical system (4-1) converges to

$$\Delta x = Ry \tag{4-4}$$

where

$$R \equiv [I - \bar{R}]^{-1} \tag{4-5}$$

so that the (reduced form) reaction coefficients are

$$R_j^j = 1/(1-\lambda^2) \qquad R_k^j = \bar{R}_k^j/(1-\lambda^2) \tag{4-6}$$

and

$$\partial R_m^\ell / \partial \bar{R}_k^j = R_j^\ell R_m^k \tag{4-7}$$

Thus in the continuous time limit when $\lambda < 1$

$$\dot{x}^j = \sum_k R_k^j y^k \tag{4-8}$$

while if $\lambda > 1$ the system explodes instantaneously. Since with even small adjustment costs an explosion is infinitely costly to all firms $\lambda \leq 1$ is assumed to act as a constraint on the system: firms will never choose to permit $\lambda > 1$.

Finally, letting S_k^j be the rate of change of $\bar{R}_k^j$ the rate of change of R_m^ℓ by (4-7) is

$$\dot{R}_m^\ell = R_j^\ell R_m^k S_k^j + R_k^\ell R_m^j S_j^k \tag{4-9}$$

Thus when $\lambda < 1$ the state equation for x remains unchanged from (1-2) while (4-9) replaces (1-3).

The frictional reaction cost function is taken to be

$$bC^j(R^j) = bm(|R_j^j - 1| + |R_k^j|) \tag{4-10}$$

which vanishes when no reaction takes place--when $\bar{R}_k^j = 0$. This is analogous to (1-7) except that it is no longer necessarily true that R_j^j is equal to one. The reason that frictional costs are taken to depend on the reduced form reaction coefficients is that it is these coefficients, rather than the structural coefficients, which magnify short-run fluctuations in autonomous output and thus determine the level of inventories that must be held.

The Local Almost Perfect Adjustment Equations:

Provided that the coefficients of section two are interpreted as reduced form coefficients, the reasoning behind the derivation of (2-13) and (2-14) describing $\dot{x}^k$ remains unchanged and

$$y^k = \tilde{f}^k \cong b\delta \mathcal{k} \sum_{\ell} \pi^k_{\ell} R^{\ell}_k \qquad (4\text{-}11)$$

is the approximate equation for autonomous output change. With symmetric initial conditions define

$$r = \bar{R}^j_k = R^j_k / R^j_j \qquad (4\text{-}12)$$

and observe that

$$R^j_j = 1/(1-r^2) \qquad (4\text{-}13)$$

Thus (4-11) reduces to

$$y^k = \frac{b\delta \mathcal{k}}{1 - r^2} \; [\pi^k_k + r\pi^k_{\ell}] \qquad (4\text{-}14)$$

Turning to S^j_k as in (2-17)

$$s_k^j = b\sum_\ell \sum_m \frac{\partial \hat{A}^j}{\partial R_m^\ell} \frac{\partial \dot{R}_m^\ell}{\partial s_k^j}$$

$$= b\left\{-\delta bm\left[\operatorname{sgn}(R_j^j-1)\frac{\partial R_j^j}{\partial s_k^j} + \operatorname{sgn}(R_k^j)\frac{\partial \dot{R}_k^j}{\partial s_k^j}\right]\right.$$

$$\left. + \delta^2 \sum_h \pi_h^j \frac{\partial}{\partial s_k^j}\left(\sum_\ell R_\ell^h y^\ell\right)\right\} \qquad (4\text{-}15)$$

$$= b\delta\{\delta\sum_h \pi_h^j \sum_\ell [b\delta \ell R_\ell^h \sum_m \pi_m^\ell R_j^m R_\ell^k + y^\ell R_j^h R_\ell^k]$$

$$- bm[R_j^j R_k^j + \operatorname{sgn}(R_k^j)(R_j^j)^2]\} \qquad (4\text{-}16)$$

where (4-8) and (4-9) are used and $\operatorname{sgn}(R_j^j-1)=1$ from (4-6) and $\lambda \leq 1$.

<u>The Steady State</u>: Reasoning as in appendix (A) shows that the region $r < 0$ is of no interest. When $r \geq 0$ (4-8) shows that

$$y^k = 0 \qquad s_k^j = 0 \qquad j,k = 1,2 \qquad j \neq k \qquad (4\text{-}17)$$

determine the position of the steady state. The condition $y^k=0$ means from (2-16) giving the profit derivatives

$$r = \frac{2 - 3q}{q} \qquad (4\text{-}18)$$

which is (3-2) in the duopoly case. Using $y^k=0$ and (4-14) to cancel terms in (4-16) and equating the resulting expression to zero gives the second equation for the steady state

$$s_k^j = b^2\delta\left\{\frac{\delta^2 k}{(1-r^2)^3}\;[r\pi_j^j + \pi_k^j]^2 - \frac{m}{(1-r^2)^2}\,[1+r]\right\} = 0 \tag{4-19}$$

Using (4-18) and (4-19) the steady state output is computed to be

$$q^{NC} = \frac{1}{4} + \sqrt{\frac{1}{16} + 2M} \tag{4-20}$$

since the second root that occurs when (4-18) and (4-19) are solved has negative levels of output for $M > 0$. Here M is as in (3-9)

$$M \equiv \frac{m}{2E^2\delta^2 k} \tag{4-21}$$

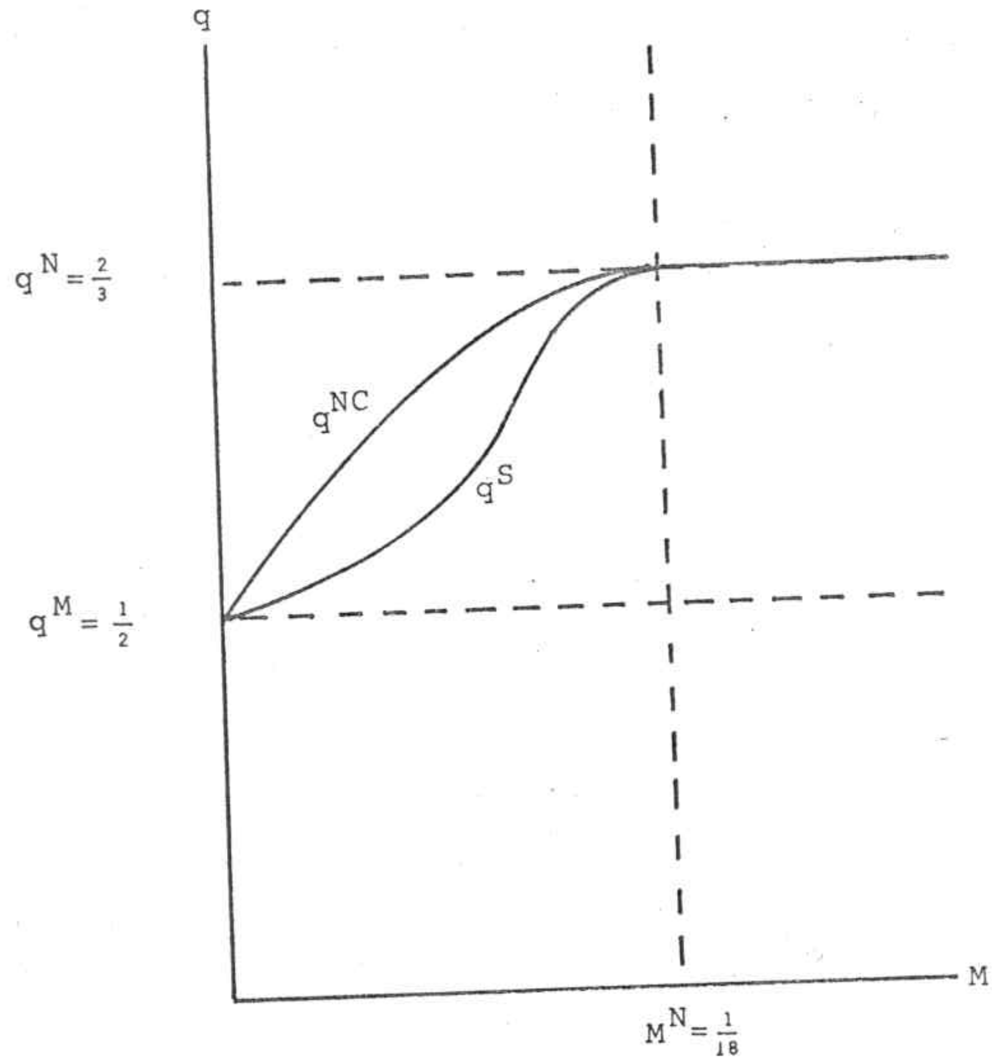
<u>Comparison to the Previous Model</u>: From section three equilibrium output when N=2 is

$$q^S = \frac{3}{4} - \sqrt{\frac{1}{16} - M} \tag{4-22}$$

by (3-7). Equating (4-20) and (4-21) shows $q^{NC}=q^S$ if and only if either M=0 in which case $q^{NC}=q^{S}=q^{M}$ or if $M=M^{N}$ in which case $q^{NC}=q^{S}=q^{N}$. Evaluating $\partial q^{NC}/\partial M = 4$ and $\partial q^{S}/\partial M = 2$ at M=0 shows that for $0 < M < M^{N}$ $q^{NC} > q^{S}$. Naturally, for $M \geq M^{N}$ output in both cases is at the Cournot-Nash level q^{N}. This situation is illustrated in figure (4-1).

In economic terms what is going on? Without communication the firms can't distinguish autonomous from total output movements. Thus they counter-react to each other's reactions. This has two effects. First, the incentives for choosing S_k^j are complicated by the fact that the rates of change of <u>all</u> the reduced form reaction coefficients are affected by S_k^j as (4-9) shows. In the previous case only $\dot{R}_k^j$ depends on S_k^j. Actually, this, while complicating (4-16) greatly, is of no economic significance: each firm can still control its rival's output by rewarding and punishing it, just as before.

The second consideration is that to achieve a given level of response as measured by R_k^j/R_k^k the sequence of counter-reactions forces R_j^j to be greater than one. The sequence of counter-reactions has the effect of magnifying reaction costs by enlarging the reduced form reactions required to enforce a particular output level along $\dot{q}=0$. Except when m=0 and there are no reaction costs to be magnified or r=0 so that no one reacts, firms have an incentive to react less strongly than in the previous section to avoid the increased marginal cost of response.



Figure(4-1): Output with and without Communication

Hence long-run output is greater without communication.

How significant is the difference between long-run output with and without communication? A calculation shows the largest case is M=(1/32) in which case $q^{NC}-q^{S}$ is 3% of competitive output. This is a modest effect relative to the gap between monopoly output q^{M} and Cournot-Nash output q^{N}, which is 17% of competitive output.

5. Facts and Myths about Oligopoly

The results of this paper contradict a number of widely believed myths about oligopoly. To conclude the paper I debunk several of these myths.

Infinite Response is Optimal: Frequently in seminars I have been told that by making sufficiently large threats against opponents a firm can get them to do anything it wants. Thus, when there are no frictional costs of response, an infinite response is optimal. Our results indicate that in the context of this paper the argument is false--the long-run steady state reaction coefficient (r) was computed equal to one. It is true in this paper that each firm controls the output of all its rivals when they follow the equilibrium adjustment procedure. The objective, however, is profit and not control of opponents output. A firm controls its rival's output only by losing control of its own output. It can drive opponents' output to zero, and perhaps even increase its market share while doing so, but to do so it must cut its own output. A large share of small profits doesn't amount to much. Of course if a firm could convince its opponents to leave the market without changing its own output it would do so--but it's hard to see how it could do so, and in the environment here it certainly can't.

A more sophisticated version of this myth holds that the way to convince opponents to leave the market is to first increase output a great deal, then begin the predatory policy outlined above. The initial increase in output insures that even when his opponents are driven from the market the predator will still have substantial sales. The reason such a policy does not work in the environment of this paper is that the potential predator's opponents are committed to retaliatory policies of their own. They will match the predator's initial output increase causing prices to drop precipitously. Although in the long run his rivals may be driven out of the market the short-run losses incurred by the predator more than offset these future gains.

If a firm can permanently commit itself to a policy of predatory threats, and if opponents adjust instantly to this policy, one firm may successfully dominate the market. In the environment here, as in real life, this possibility cannot occur.

Negative Response is Optimal: This myth might best be labelled the Cournot-Nash confusion. The assertion is that when an opponent increases output it is optimal to reduce output. In this paper, however, the converse is true--all stable steady states have non-negative response rates. Why is this?

Lowering output in response to an opponent's increase has two effects: it increases profits, and it encourages the rival firm to increase output even further. At a steady state only the latter effect matters--rival firms aren't going to change output unless encouraged to do so. The Cournot-Nash confusion ignores the fact that retaliation affects the behavior of opponents. Only when it does not should opponents output increases be met with reductions.

Negotiation Matters: The institutional industrial organizational literature[15] frequently distinguishes between explicit collusion where firms negotiate output shares and implicit collusion where they do not. It argues that in the former case firms will always collude fully because it is "jointly optimal" to do so. Of course if firms can enter into legally binding contracts which can be costlessly enforced, this might be true, but is it true without legally binding contracts? The error does not lie in believing negotiation matters. Communication can (as we saw in section four) enhance collusion. It is also possible that explicit negotiations can reduce frictional costs of enforcing collusive arrangements. But does explicit collusion invalidate the results of this paper?

The key question is: how is collusion enforced? Even if an agreement is reached what keeps firms from cheating on it? Once an agreement is reached, firms will cheat and other firms retaliate--this process of learning and enforcement is what this paper is all about. A negotiated agreement could determine the starting point for the dynamical system described here, but those equations of motion will determine the history of the industry from that point on. It is a mistake to think that talk alone will cure the problem of enforcing collusive arrangements.

Facts about Oligopoly: Learning is a time-consuming activity. Because it takes time to reoptimize, firms are implicitly committed to the status quo. This paper has examined how firms (almost) rationally choose these implicit commitments. The result is a simple sensible story of oligopoly: firms punish uncooperative opponents and reward cooperative ones. In the long-run steady state this implies that market competitiveness depends on the frictional costs of enforcing collusive arrangements.