Repeated Games

Long Run versus Short Run Player

a fixed simultaneous move *stage game*

Player 1 is long-run with discount factor δ

actions $a^1 \in A^1$ a finite set

utility $u^1(a^1, a^2)$

Player 2 is short-run with discount factor 0

actions $a^2 \in A^2$ a finite set

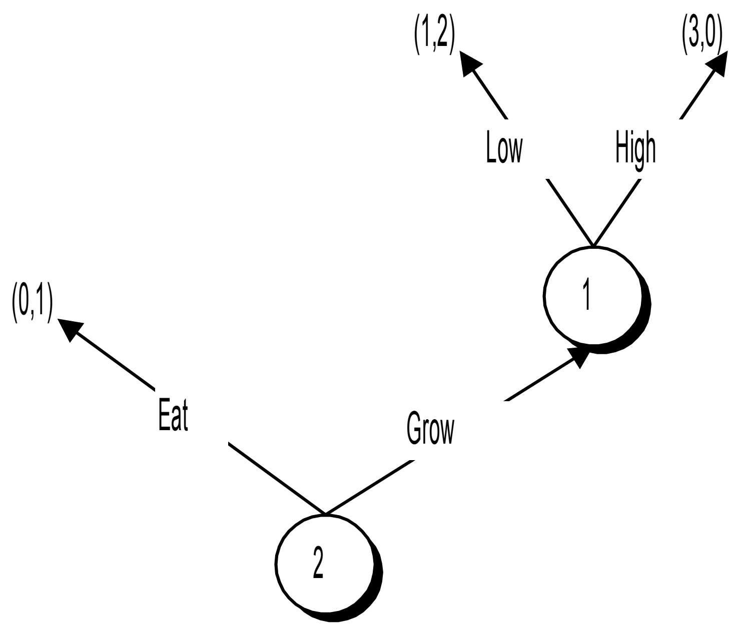
utility $u^2(a^1, a^2)$

What it is about

the “short-run” player may be viewed as a kind of “representative” of many “small” long-run players

- ♦ the “usual” case in macroeconomic/political economy models
- ♦ the “long run” player is the government
- ♦ the “short-run” player is a representative individual

Example 1: Peasant-Dictator



Example 2: Backus-Driffil

	Low	High
Low	0,0	-2,-1
High	1,-1	-1,0

Inflation Game: LR=government, SR=consumers

consumer preferences are whether or not they guess right

	Low	High
Low	0,0	0,-1
High	-1,-1	-1,0

with a hard-nosed government

Repeated Game

history $h_t = (a_1, a_2, \ldots, a_t)$

null history h_0

behavior strategies $\alpha_t^i = \sigma^i(h_{t-1})$

long run player preferences

average discounted utility

$$(1-\delta)\sum_{t=1}^{T} \delta^{t-1} u^i(a_t)$$

note that average present value of 1 unit of utility per period is 1

Equilibrium

Nash equilibrium: usual definition – cannot gain by deviating

Subgame perfect equilibrium: usual definition, Nash after each history

Observation: the repeated static equilibrium of the stage game is a subgame perfect equilibrium of the finitely or infinitely repeated game

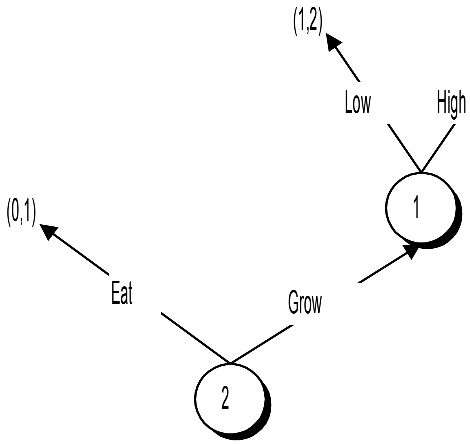
♦ strategies: play the static equilibrium strategy no matter what

"perfect equilibrium with public randomization"

may use a public randomization device at the beginning of each period to pick an equilibrium

key implication: set of equilibrium payoffs is convex

Example: Peasant-Dictator



normal form: unique Nash equilibrium **high, eat**

	eat	grow
low	0*,1	1,2*
high	0*,1*	3*,0

Static Benchmarks

payoff at static Nash equilibrium to LR player: 0

precommitment or Stackelberg equilibrium

precommit to low get 1

mixed precommitment to 50-50 get 2

minmax payoff to LR player: 0

Payoff Space

utility to long-run player

mixed precommitment/Stackelberg = 2

best dynamic equilibrium = ?

pure precommitment/Stackelberg = 1

Set of dynamic equilibria

static Nash = 0

worst dynamic equilibrium = ?

minmax = 0

Repeated Peasant-Dictator

finitely repeated game

final period: high, eat, so same in every period

Do you believe this??

♦ Infinitely repeated game

begin by low, grow

if low, grow has been played in every previous period then play low, grow

otherwise play high, eat (reversion to static Nash)

claim: this is subgame perfect

When is this an equilibrium?

clearly a Nash equilibrium following a history with high or eat

SR play is clearly optimal

for LR player

may high and get $(1-\delta)3+\delta 0$

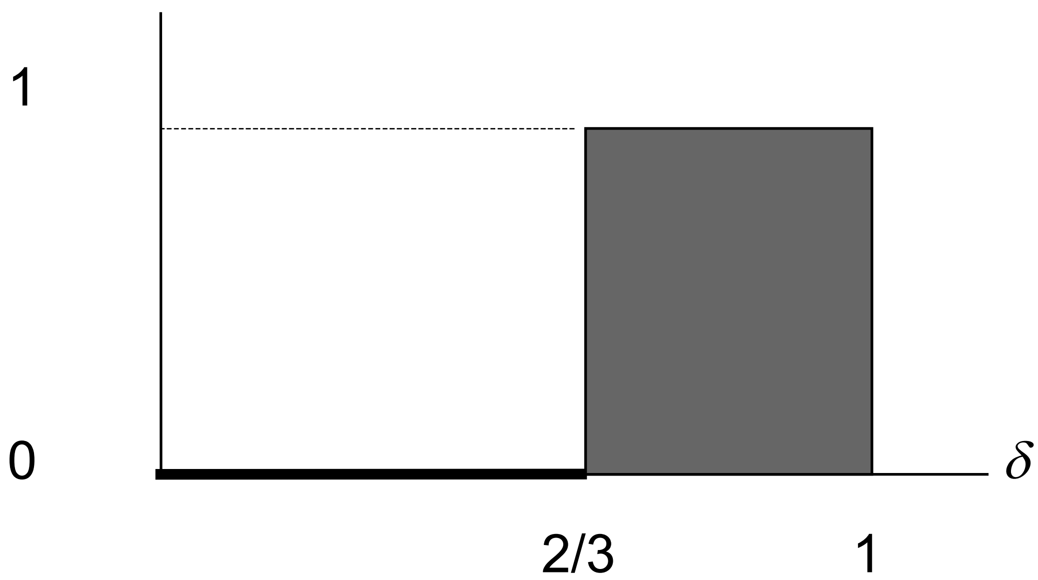
or low and get 1

so condition for subgame perfection

$(1-\delta)3 \leq 1, \delta \geq 2/3$

Equilibrium Utility

equilibrium utility for LR



General Deterministic Case

Fudenberg, Kreps and Maskin

- max $u^1(a)$
- mixed precommitment/Stackelberg
- $\bar{v}^1$ best dynamic equilibrium
- pure precommitment/Stackelberg
- Set of dynamic equilibria
- static Nash
- $\underline{v}^1$ worst dynamic equilibrium
- minmax
- min $u^1(a)$

Characterization of Equilibrium Payoff

$\alpha = (\alpha^1, \alpha^2)$ where α^2 is a b.r. to α^1

α represent play in the first period of the equilibrium

$w^1(a^1)$ represents the equilibrium payoff beginning in the next period

$$v^1 \geq (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1)$$

$$v^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1), \alpha^1(a^1) > 0$$

$$\underline{v}^1 \leq w^1(a^1) \leq \overline{v}^1$$

Simplified Approach

impose stronger constraint using n static Nash payoff

for best equilibrium $n \leq w^1(a^1) \leq \overline{v}^1$

for worst equilibrium $\underline{v}^1 \leq w^1(a^1) \leq n$

avoids problem of best depending on worst

remark: if we have static Nash = minmax then no computation is neede for the worst, and the best calculation is exact.

max problem

fix $\alpha = (\alpha^1, \alpha^2)$ where α^2 is a b.r. to α^1

$$\bar{v}^1 \geq (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1)$$

$$\bar{v}^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1), \alpha^1(a^1) > 0$$

$$n^1 \leq w^1(a^1) \leq \bar{v}^1$$

how big can $w^1(a^1)$ be in = case?

Biggest when $u^1(a^1, \alpha^2)$ is smallest, in which case

$$w^1(a^1) = \bar{v}^1$$

$$\bar{v}^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta \bar{v}^1$$

Summary

conclusion for fixed α

$$\min_{a^1|\alpha(a^1)>0} u^1(a^1,\alpha^2)$$

i.e. worst in support

$$\bar{v}^1 = \max_{\alpha^2 \in BR^2(\alpha^1)} \min_{a^1|\alpha(a^1)>0} u^1(a^1,\alpha^2)$$

observe:

mixed precommitment $\geq \bar{v}^1 \geq$ pure precommitment

Peasant-Dictator Example

	eat	grow
low	0*,1	1,2*
high	0*,1*	3*,0

p(low)	BR	worst in support
1	grow	1
$\frac{1}{2}<p<1$	grow	1
p=1/2	any mixture	≤ 1(low)
$0<p<\frac{1}{2}$	eat	0
p=0	eat	0

Check the constraints

$$w^1(a^1) = \frac{\bar{v}^1 - (1-\delta)u^1(a^1, \alpha^2)}{\delta} \geq n^1$$

as $\delta \to 1$ then $w^1(a^1) \to \bar{v}^1 \geq n^1$

min problem

fix $\alpha = (\alpha^1, \alpha^2)$ where α^2 is a b.r. to α^1

$$\underline{v}^1 \geq (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1)$$
$$\underline{v}^1 \leq w^1(a^1) \leq n^1$$

Biggest $u^1(a^1, \alpha^2)$ must have smallest $w^1(a^1) = \underline{v}^1$

$$\underline{v}^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta \underline{v}^1$$

conclusion

$$\underline{v}^1 = \max u^1(a^1, \alpha^2)$$

or

$\underline{v}^1 = \min_{\alpha^2 \in BR^2(\alpha^1)} \max u^1(a^1, \alpha^2)$, that is, constrained minmax

Worst Equilibrium Example

	L	M	R
U	0,-3	1,2	0,3
D	0,3*	2,2	0,0

static Nash gives 0

minmax gives 0

worst payoff in fact is 0

pure precommitment also 0

mixed precommitment

p is probability of up

to get more than 0 must get SR to play M

$-3p+(1-p)3 \le 2$ and $3p \le 2$

first one

$-3p+(1-p)3 \le 2$

$-3p-3p \le -1$

$p \ge 1/6$

second one

$3p \le 2$

$p \le 2/3$

want to play D so take $p = 1/6$

get $1/6+10/6=11/6$

Utility to long-run player

max $u^1(a)$=2

mixed precommitment/Stackelberg=11/16

$\overline{v}^1$ best dynamic equilibrium=1

pure precommitment/Stackelberg=0

Set of dynamic equilibria

static Nash=0

$\underline{v}^1$ worst dynamic equilibrium=0

minmax=0

min $u^1(a)$=0

calculation of best dynamic equilibrium payoff

p is probability of up

p	BR^2	worst in support
<1/6	L	0
1/6<*p<5/6*	M	1
p>5/6	R	0

so best dynamic payoff is 1