You have three hours. You should do all four questions. Each question has equal weight. It is recommended that you read the entire exam before doing any questions.

1. Static Concepts of Equilibrium

In the game with payoff matrix

0,4	0,4
-1,1	1,3

- Find all Nash equilibria pure and mixed.
- If player one can precommit what is the optimal pure and mixed precommitment and corresponding payoffs?

2. Trembling Hand Perfection

A strategy profile σ is *trembling hand perfect* if there exists a sequence of strategy profiles $\sigma^n \to \sigma$ with $\sigma_i^n(s_i) > 0$ for all i and $s_i \in S_i$ such that $\sigma_i(s_i) > 0$ implies that s_i is a best-response to σ_{-i}^n. Prove that every trembling hand perfect profile is a Nash equilibrium. Give an example of a Nash equilibrium in a 2x2 game which is not trembling hand perfect and explain why.

3. Equilibrium in a Repeated Game

Consider the simultaneous move stage game:

	U	D
U	6,6	-1,10
D	10,-1	0,0

Consider the "grim" strategy of playing U in period one, playing U as long as both players have played U in the past, and playing D otherwise. For what discount factors δ do these strategies form a subgame perfect equilibrium?

4. The Chain Store Paradox Paradox

Consider the Kreps-Wilson version of the chain store paradox: An entrant may stay out and get nothing (0), or he may enter. If he enters, the incumbent may fight or acquiesce. The entrant gets b if the incumbent acquiesces, and $b-1$ if he fights, where $0 < b < 1$. There are two types of incumbent, both receiving $a > 1$ if there is no entry. If there is a fight, the strong incumbent gets 0 and the weak incumbent gets -1; if a strong incumbent acquiesces he gets -1, a weak incumbent 0.

Only the incumbent knows whether he is weak or strong; it is common knowledge that the entrant a priori believes that he has a π_0 chance of facing a strong incumbent. Define

$$\gamma = \frac{p_0}{1-p_0}\frac{1-b}{b}$$

a. Sketch the extensive form of this game.

b. Define a sequential equilibrium of this game.

c. Show that if $\gamma \neq 1$, there is a unique sequential equilibrium, and that if $\gamma > 1$ entry never occurs, while if $\gamma < 1$ entry always occurs.

d. What are the sequential equilibria if $\gamma = 1$?

e. Now suppose that the incumbent plays a second round against a different entrant who knows the result of the first round. The incumbent's goal is to maximize the sum of his payoffs in the two rounds. Show that if $\gamma > 1$ there is a sequential equilibrium in which the entrant enters on the first round and both types of incumbents acquiesce. Be careful to specify both the equilibrium strategies and beliefs.