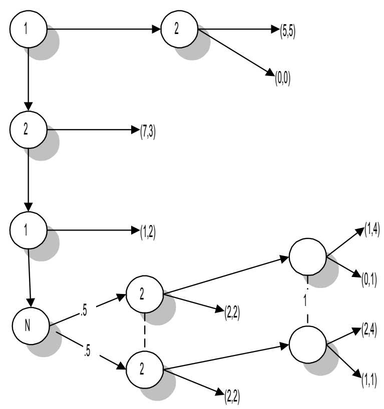
Problems on Dyamic Games

1. Extensive Form

In the game below find 1) the normal form 2) all pure and mixed Nash equilibria 3) all subgame perfect equilibra



2. Backward Induction

There are five pirates with names 1,2,3,4,5. They have just seized a hundred gold coins, and now it's time to share the loot. The bargaining rules are: Whoever has the lowest number as a name must propose an division of the one hundred coins to the remaining pirates. If the majority accepts the proposal, then the coins are allocated and the game ends. If the majority does not accept, then the proposer gets thrown overboard and the game is repeated with one less pirate. What should the first pirate propose?

3. Self-Confirming Equilibrium

Consider a three person centipede game in which player 1 can drop or pass, player 2 can drop or pass, and player 3 can drop or pass. If player 1 drops, the payoffs are (5,3,5); if player 2 drops the payoffs are (4,5,4), if player 3 drops the payoffs are (3,4,3) and if player 3 passes the payoffs are (8,6,8). What payoffs are possible in Nash equilibrium? In sequential equilibrium? Construct a heterogeneous self-confirming equilibrium that is NOT a public randomization over Nash equilibrium.

4. Chain Store Game

Consider the following chain store game played between a patient player one (chain store) with discount factor δ and a sequence of short-run myopic player 2's (entrants – with discount factor 0)

	out	in
fight	3,0	-2,-2
give in	4,0	2,2

a. What is the Nash equilibrium if the game is played once?

b. What is the Stackelberg equilibrium in which player 1 gets to commit if the game is played once?

c. What is the subgame perfect equilibrium if the game is repeated $T < \infty$ times?

d. If the game is infinitely repeated, find a δ and strategies for both players such that the long-run player gets 3.

5. Brazil or the U.S.?

A long-lived government faces a short-run representative government employee. The government must choose whether to honor pensions (H) or not (N). At the beginning of the period, times are either "good" or "bad." The probability times are "bad" is 90%. In good times, pensions are always honored. In bad times they are honored or not depending on the government decision. The employee is informed and observes (after the fact, at the end of the period) whether or not times are good or bad. The choice of the employee is to guess whether or not her pension will be honored (H) or (N). The payoff of the employee is the sum of two parts: 1 if the pension is honored, 0 if it is not; and 1 for guessing right, 0 for guessing wrong. So guessing right when the pension is honored gives 2, and so forth. Government payoffs in both good and bad times are

	Guess H	**Guess N**
H	2	0
N	3	1

a. Find the extensive and normal forms of the stage-game.

b. For the long-run player, find the minmax, the static Nash, mixed precommitment and pure precommitment payoffs.

c. Find the worst equilibrium for the long-run player, and describe in general terms the set of equilibrium payoffs for the long-run player.

d. How patient must the government be to avoid catastrophe?

5. Trembling Hand Perfection

A strategy profile σ is *trembling hand perfect* if there exists a sequence of strategy profiles $\sigma^n \to \sigma$ with $\sigma_i^n(s_i) > 0$ for all i and $s_i \in S_i$ such that $\sigma_i(s_i) > 0$ implies that s_i is a best-response to σ_{-i}^n . Prove that every trembling hand perfect profile is a Nash equilibrium. Give an example of a Nash equilibrium in a 2x2 game which is not trembling hand perfect and explain why.

6. Subgame Perfection, Self-Confirming Equilibrium

by Tuna Docmeci based on Osborne and Rubinstein Exercise 103.2 and 2021 exam

Two players play the following game. The first player moves first, and can end the game, resulting in each player getting a payoff of 1. She can decide to continue the game, in which case a simultaneous move game is played where each player announces a non negative integer. The payoff of each player is then the product of the two numbers.

a) Write down the extensive form, specifying the action spaces at each stage. Find the subgame perfect Nash equilibrium of this game.

b) Now suppose that if the game is continued, the players can announce an integer in the interval [0,10]. Construct a self-confirming equilibrium where both players get 50.5 in expectation. Make sure to write down on and off-path beliefs, and strategies.

7. Tacit Collusion

Tuna Docmeci

N firms compete in a market of homogeneous goods. They compete in prices: they set the prices and meet the demand. Demand is given by $D(p)$, $D'(p) < 0$, and there exists a $\bar{p} < \infty$ such that $\forall p \geq \bar{p}, D(p) = 0$. Normalize the unit cost of production c to 0. The demand for firm i is: 1) $D(p_i)$ if it charges the lowest price in the market, 2) $D(p_i)/M$ if $M - 1$ other firms are charging the lowest price together with firm i, and 3) 0 if at least one competitor is charging a lower price than firm i.

a) Find the Nash equilibrium of the stage game where each firm maximizes the profit.

b) Now suppose that the firms interact in the market for T periods, and maximize the stream of profits: $\sum_{t=0}^{T} \delta^t \pi_{it}(p_i t, \mathbf{p}_{-it})$. Use backward induction to find the unique SPNE.

c) Now firms compete over infinite horizon and maximize

$$\sum_{t=0}^{\infty} \delta^t \pi_{it}(p_i t, \mathbf{p}_{-it}).$$

Consider the following equilibrium: Firms charge $p_h > 0$ if $t = 1$ or if all other firms have charged p_h in the past, and charge 0 otherwise. For what discount factors is this a subgame perfect equilibrium, and collusion is sustainable?

8. An Insurrection Game

A long run government with discount factor δ faces an infinite sequence of one-period lived insurrectionists. The government may either fight or appease and the insurrectionists can either attack or stay home. The insurrectionists get zero for staying home, they get one if they attack and are appeased and minus one if they attack and the government fights. The government fights when there is no insurrection they must pay one to cover the costs of putting troops on the street. If they fight when there is an insurrection the cost is three. If they do not fight and there is no insurrection they get zero, but if there is an insurrection they get minus two.

a. What is the unique Nash equilibrium if the game is played once?

b. What is the highest possible subgame perfect equilibrium payoff for the government in the repeated game and for what discount factor is this in fact an equilibrium?