Microeconomics 2, Problem Set 1 Solutions

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Exercise 1: For each of the following games find 1) all weak and strict dominant strategy equilibria 2) apply iterated strict dominance 3) find all pure and mixed Nash equilibria 4) indicate which Nash equilibria are trembling hand perfect and why

The underlined payoffs in the tables indicate the payoff of a player from her best response to the corresponding strategy of the opponent.

Solution:
1) a)

Table 1:

	L	R
U	$\underline{2},\underline{1}$	0,0
D	0,0	$\underline{1},\underline{2}$

1) Dominant Strategy Equilibrium: Note that a strictly (weakly) dominant strategy σ_i of i is a strategy that is the unique best response (in the best response set) for any given strategy profile σ_{-i} of the other players. A dominant strategy equilibrium is a strategy profile where each Player is playing a dominant strategy.

For Player 1, we have that the unique best response to L is U $(2 > 0)$ and the unique best response to R is D $(1 > 0)$. Thus, there are no strictly or weakly dominant strategies for Player 1 (same holds for Player 2). Thus, there are no equilibria in strictly or weakly dominant strategies.

Furthermore, the best responses shown above imply that neither player has strictly *dominated* strategies (as each pure strategy is a best response to some belief). Thus, no strategy can be deleted by iterated strict dominance.

When looking for pure Nash Equilibria(NE), note that it is sufficient to consider deviations to other pure strategies. This is because the payoff from mixing against a given opponent strategy result in a convex combination of the pure strategy payoffs with positive probability. In the above game we have:

$$u_1(U,L) = 2 > 0 = u_1(D,L) \Rightarrow \rho_1(L) = \{U\}$$
$$u_2(U,L) = 1 >= 0 = u_2(U,R) \Rightarrow \rho_2(U) = \{L\}$$
$$u_1(D,R) = 1 > 0 = u_1(U,R) \Rightarrow \rho_1(R) = \{D\}$$
$$u_2(D,R) = 2 > 0 = u_2(D,L) \Rightarrow \rho_2(D) = \{R\}$$

Where $\rho_i(\sigma_{-i})$ is the best response of i to strategy σ_{-i}. This means that the two pure strategy equilibria are {U,L} and {D,R}. For a mixed strategy equilibrium, we need that both players are indifferent between all pure actions in the support of the mixed strategy (given the strategy of the opponent). If one pure strategy in the support is strictly preferred to the other, one could always improve the expected payoff by increasing the weight on that pure strategy. So unless there's an indifference between all pures that are being played with positive probability, a mixed strategy cannot be best-response.

Here we have the indifference condition for σ_2^*:

$$u_1(U,\sigma_2^*) = 2\sigma_2^*(L) = \sigma_2^*(R) = u_1(D,\sigma_2^*)$$
$$\Rightarrow \sigma_2^* = \{\sigma_2^*(L); \sigma_2^*(R)\} = \{\frac{1}{3}; \frac{2}{3}\}$$

and for σ_1^*:

$$u_2(\sigma_1^*,L) = \sigma_1^*(U) = 2\sigma_1^*(D) = u_2(\sigma_1^*,R)$$
$$\Rightarrow \sigma_1^* = \{\sigma_1^*(U); \sigma_1^*(D)\} = \{\frac{2}{3}; \frac{1}{3}\}$$

Where $\sigma_i(s_i)$ is the probability that strategy σ_i assigns to pure strategy s_i for Player i. So strategy profile $\sigma = \{\sigma_1^*; \sigma_2^*\}$ as described above is the mixed NE.

Finally, see Exercise 5 for the formal definition of trembling hand perfection. We know that every strict or fully mixed equilibria are THP. Here, since any deviation from the pure NEs yield strictly lower payoff and there are only 2 pures for each player (which implies that the mixed NE is fully mixed as both pures are played with positive probability), all NEs are THP.

For demonstration, take the fully mixed sequence

$$\sigma^n = \{\sigma_1^n(U), \sigma_2^n(L)\} = \{(1 - \frac{1}{3n}), (1 - \frac{1}{3n})\}$$

This sequence converges to $\{U,L\}$ as $n \to \infty$, and the best responses are $\rho_1(\sigma_2^n) = U$ and $\rho_2(\sigma_1^n) = D$ for all $n \in \mathbb{N}$. This satisfies the condition for THP for $\{U,D\}$. For mixed NE σ^*, take the sequence $\sigma^n = \sigma^*$ for all $n \in \mathbb{N}$. This constant sequence trivially converges to σ^* and yields the same best responses.

1) b)

Table 2:

	L	R
U	6,6	0,$\underline{7}$
D	$\underline{7}$,0	$\underline{1}$,$\underline{1}$

This game is the standard Prisoner's Dilemma. Since $u_1(D, L) = 7 > 6 = u_1(U, L)$ and $u_1(D, R) = 1 > 0 = u_1(U, R)$, we have that D is the (strictly) dominant strategy for Player 1 (dominates the only other pure strategy). Similarly, since $u_2(U, R) = 7 > 6 = u_2(U, L)$ and $u_2(D, R) = 1 > 0 = u_2(D, L)$, R is the (strictly) dominant strategy for Player 2. Thus, the strict dominant strategy equilibrium is $\{D, R\}$.

As shown above, D strictly dominates U, and R strictly dominates L. We can remove the dominated strategies in any order to be left with the only surviving strategy profile $\{D, R\}$. A game where a unique strategy profile survives the iterated elimination of strict dominance is also called *dominance solvable*.

If a game is dominance solvable, it has a unique NE. This follows directly from the first statement in Excercise 2: *A profile is a NE in the original game if and only if it is a NE of the game remaining after iterated strict dominance*. Thus we have a unique (mixed or pure) NE in this game, namely $\{D, R\}$.

Once again, as any deviation from this unique equilibrium yields strictly lower payoff, this is a strict NE. Since it is a strict NE, it is THP.

1) c)

Table 3:

	L	C	R
U	$\underline{3}$,$\underline{3}$	2,2	$\underline{1}$,1
M	2,2	1,1	0,$\underline{8}$
D	1,$\underline{1}$	$\underline{8}$,0	0,0

Writing the best response sets of Player 1 to 2's pure strategies, we have that:

$$\rho_1(L) = \rho_1(R) = \{U\}, \rho_1(C) = \{D\}$$

Since weak or strict dominance would imply that the dominant strategy be in best response sets to all opponent strategies, Player 1 here does not have a dominant strategy. From this we can conclude that this game doesn't yield an equilibrium in dominant strategies.

However, it is possible to iteratively eliminate strictly dominated strategies. Note that for Player 1, U str. dominates M. Once we eliminate M, L dominates both C and R in the remaining game. Now that the only remaining strategy of Pl2 is L, the best response of Pl1 is U. Thus, the only surviving profile is $\{U, L\}$.

Once again, since the game is dominance solvable, the only NE of the game is $\{U, L\}$, and since any deviation from $\{U, L\}$ yileds strictly lower payoffs, it is THP.

1) d)

Table 4:

	L	R
U	$\underline{1},\underline{3}$	$1,\underline{3}$
D	$0,\underline{0}$	$\underline{2},\underline{0}$

Here, note that $u_2(\sigma_1, L) = u_2(\sigma_1, R)$ for all possible $\sigma_1 \in \Delta\{U, D\}$. That, is Player 2 is indifferent between all of her strategies given any possible strategy of Player 1. Thus, any possible strategy $\sigma_2 \in \Delta\{L, R\}$ of Player 2 is weakly dominant, and no strategy is strictly dominant. However for Player 1 we have that $\rho_1(L) = \{U\}$ and $\rho_1(R) = \{D\}$. Thus, Player 1 has no dominant strategies (weak or strict), and this game does not yield dominant strategy equilibria.

Since it is shown above that neither player has strictly dominated strategies, iterated elimination is not possible.

For the NEs of this game, recall that Player 2 is always indifferent between all of her possible strategies. Thus, any strategy Player 2 plays will be a best response, regardless of what Player 1 plays. Then, the NE strategy profiles are exactly those where σ^* where σ_1^* is a best response to σ_2^*. The best response function of Player 1 is:

$$\rho_1(\sigma_2) = \begin{cases} U, & \sigma_2(L) < \frac{1}{2} \\ \Delta\{U, D\}, & \sigma_2(L) = \frac{1}{2} \\ D, & \sigma_2(L) > \frac{1}{2} \end{cases}$$

Then we can formally define the set of NE profiles as:

$$\Sigma^* := \left\{ \sigma \in \Sigma | (\sigma_1 = U \wedge \sigma_2(L) > \frac{1}{2}) \vee (\sigma_1 = D \wedge \sigma_2(L) < \frac{1}{2}) \vee (\sigma_2 = \frac{1}{2}) \right\}$$

Where Σ is the set of all strategy profiles in the game. When looking for THP, we can consider different subsets of the above NE set.

First, consider those equilibria σ^* with $\sigma_1^* = U$ and $\sigma_2^*(L) > \frac{1}{2}$. We can specify a fully mixed sequence $\sigma^n = \{\sigma_1^n(U), \sigma_2^n(L)\} = \{1 - \epsilon_n, \sigma_2^*(L) - \epsilon_n\}$ with ϵ_n converging to zero and $\epsilon_n \in (0, \sigma_2^*(L) - 1/2)$ for all $n \in \mathbb{N}$. We then have that σ^n converges to σ^*. Furthermore, since $\sigma_2^n \geq \frac{1}{2}$ for all n, U is always a best response of Player 1 to σ_2^n. Thus, these equilibria are THP.

When we consider equilibrium $\sigma_1^* = U$ with $\sigma_2^*(L) = \frac{1}{2}$, we can use the sequence $\{\sigma_1^n(U), \sigma_2^n(L)\} = \{1 - \epsilon_n, \sigma_2^*(L)\}$ with the same properties for ϵ_n as the previous case.

The properties satistied for the set of equilibria with $\sigma_1^* = D$ and $\sigma_2^*(L) < \frac{1}{2}$ with sequence $\sigma^n = \{\sigma_1^n(U), \sigma_2^n(L)\} = \{\epsilon_n, \sigma_2^*(L) - \epsilon_n\}$ with ϵ_n converging to 0 and $\epsilon_n \in (0, \sigma_2^*(L))$ for all $n \in \mathbb{N}$. These equilibria are also THP.

When we consider equilibrium $\sigma_1^* = D$ with $\sigma_2^*(L) = \frac{1}{2}$, we can use the sequence $\{\sigma_1^n(U), \sigma_2^n(L)\} = \{\epsilon_n, \sigma_2^*(L)\}$ with the same properties for ϵ_n as the previous case.

The Nash equilibria that remain are with $\sigma_1^* \in (0, 1)$ and $\sigma_2^* = \frac{1}{2}$. As these NEs are fully mixed, they are THP.

Exercise 2:

Prove that a profile is a Nash equilibrium of a game if and only if it is the Nash equilibrium of the game in which strategies have been removed by iterated strict dominance. Prove that a Nash equilibrium of a game in which strategies have been removed by iterated weak dominance is a Nash equilibrium of the original game. Give an example of a Nash equilibrium of a game that is not a Nash equilibrium of the game where strategies have been removed by iterated weak dominance.

Solution: Let Σ be the set of strategy profiles in the initial game and Σ^N be the set of profiles that survive iterated strict dominance after $N \in \mathbb{N}$ iterations (implies $\Sigma^N \subseteq \Sigma$).

a)

Claim: σ^* NE in $\Sigma \iff \sigma^*$ is a NE in Σ^N.

Proof:

$\Rightarrow$: Let σ^* be a NE in Σ. For this direction of the implication, we must show two things:

1) σ^* is a NE in Σ and $\sigma^* \in \Sigma^N \Rightarrow \sigma^*$ is a NE in Σ^N.

2) A NE profile can't be eliminated by iterated strict dominance

1) Follows directly from $\Sigma^N \subseteq \Sigma$. If there are no profitable deviations from σ^* in Σ, there can't be any profitable deviations in any subset of Σ.

2) Suppose $\sigma^* \notin \Sigma^N$. In this case, some strategy σ_i^* from profile σ^* has to be eliminated. Then in some subset Σ^n, of Σ (with $n < N$), there exists some $\sigma_i^{'}$ that strictly dominates σ_i^*. By definition of strict dominance, this implies:

$$u_i(\sigma_i', \sigma_{-i}^*) > u_i(\sigma^*)$$

However, this is a profitable deviation from σ_i^* within Σ_i, which contradicts that σ^* is a NE of the initial game.

$\Leftarrow$: Let σ^* be a NE in Σ^N. Suppose $\sigma_i' \in \Sigma_i / \Sigma_i^N$ is a profitable deviation for i from σ_i^*. That is, $u_i(\sigma_i', \sigma_{-i}^*) > u_i(\sigma^*)$.

Now since $\sigma_i' \notin \Sigma_i^N$, it must be strictly dominated by some strategy σ_i'' in some stage Σ^n with $n < N$. Since $\sigma_{-i} \in \Sigma_{-i}^N$ (that is, we know that the opponents' equilibrium strategies survived), we know that $\sigma_{-i}^* \in \Sigma_{-i}^n$. Then strict dominance at stage n implies that $u_i(\sigma_i'', \sigma_{-i}^*) > u_i(\sigma_i', \sigma_{-i}^*)$. However, unless $\sigma_i'' \in \Sigma^N$, the same will apply to σ_i'' at a later iteration (and so forth).

Since N is finite, finitely many iterations of this argument (at most $N - n$ times), will lead to $u_i(\tilde{\sigma}_i, \sigma_{-i}^*) > u_i(\sigma_i', \sigma_{-i}^*)$ for some $\tilde{\sigma}_i \in \Sigma_i^N$. Finally, since σ^* is a NE in Σ^N, it must be weakly better for i than any other strategy in Σ_i^N. This implies $u_i(\sigma^*) \geq u_i(\tilde{\sigma}_i, \sigma_{-i}^*) > u_i(\sigma_i', \sigma_{-i}^*)$, which contradicts that σ_i' is a profitable deviation from σ_i^*.

b)

This claim is proven for weak iteration with the same argument as the second direction in part **a)**, except that the argument that the strategies outside the final game are dominated should be made with weak inequalities.

Note that the converse doesn't hold in this case since NEs of the original game can be eliminated by weak iteration as shown below.

c)

Consider the following game.

Table 5:

	L	C	R
U	$\underline{1},\underline{2}$	0,1	0,0
M	$\underline{1}$,0	$\underline{2},\underline{2}$	0,0
D	0,0	0,0	$\underline{1},\underline{1}$

Here, there are 3 pure NEs: $\{U, L\}, \{M, C\}$ and $\{D, R\}$. However, since U is weakly dominated by M, iterated weak elimination would rule out $\{U, L\}$.

Exercise 3:

Consider the game

Table 6:

	A_2	B_2
A_1	0,0	$\underline{2},\underline{1}$
B_1	$\underline{1},\underline{2}$	0,0

Show that the correlated strategy profile is in fact a correlated equilibrium.

Table 7:

	A_2	B_2
A_1	1/3	1/3
B_1	1/3	0

Solution:

For the above game, consider a public randomization mechanism with states $\Omega = \{\omega_1, \omega_2, \omega_3\}$, each occurring with equal probability. Now let the information partitions (i.e. set of signals) for the players be $\Omega_1 = \{\{\omega_1, \omega_2\}, \omega_3\}$ and $\Omega_2 = \{\omega_1, \{\omega_2, \omega_3\}\}$. So Player 1 cannot distinguish between state 1 or 2 and Player 2 cannot distinguish between states 2 and 3. A strategy of player i is then a mapping $s_i : \Omega_i \to \{A_i, B_i\}$. Now consider the following strategy profile:

$$s_1(\omega) = \begin{cases} A_1, & \omega = \{\omega_1, \omega_2\} \\ B_1, & \omega = \omega_3 \end{cases}$$

$$s_2(\omega) = \begin{cases} A_2, & \omega = \{\omega_2, \omega_3\} \\ B_2, & \omega = \omega_1 \end{cases}$$

Let $\rho_i(s_{-i};\omega)$ be the best response of i to opponent given signal ω. Then from above, we have $\rho_1(s_2;\{\omega_1,\omega_2\}) = \rho_1(\frac{1}{2}A_2, \frac{1}{2}B_2) = A_1$. That is, upon observing the signal that the state is ω_1 or ω_2 (with equal probability), Player 1 infers that under her strategy, Player 2 will play A_2 or B_2 equally likely (A_2 if ω_2 and B_2 if ω_1). Similarly we have $\rho_1(s_2;\omega_3) = B_1$ (since with ω_3, s_2 yields A_2 for sure). For Player 2, $\rho_2(_1,\omega_1) = \rho_2(A_1) = B_2$ and $\rho_2(s_1,\{\omega_2,\omega_3\}) = \rho_2(\frac{1}{2}A_1, \frac{1}{2}B_1\} = A_2$. Each player is playing best response to the others' strategy conditional on the signal they receive. So this strategy profile constitutes a NE.

Finally note that with these strategies, ω_1 implies $\{A_1, B_2\}$, ω_2 implies $\{A_1, A_2\}$, and ω_3 implies $\{B_1, A_2\}$. As states occur with 1/3 probability each, we have the distribution over outcomes that we were looking for.

Exercise 4:

Two players must choose whether to specialize – they must choose between being a hunter and a gatherer. After they choose, they meet to play a game. If both are hunters, or both are gatherers, they get no benefit from specialization, and receive a utility of zero. If one is a hunter and one a gatherer, the hunter receives 2 and the gatherer 1 unit of utility. 1) Write the normal form of the game. 2) Find the symmetric Nash equilibrium in which both players employ the same strategy. 3) Find a symmetric correlated equilibrium (probabilities remain the same when we interchange rows for columns) which Pareto dominates the symmetric Nash equilibrium. The correlated equilibrium may use public randomization if you wish, but you must show it is a correlated equilibrium by showing that neither player wishes to deviate from the recommendation of the randomization device.

Solution:

The normal form of the game yields:

Table 8:

	H_2	G_2
H_1	0,0	$\underline{2},\underline{1}$
G_1	$\underline{1},\underline{2}$	0,0

Which is the same strategic form game as in exercise 3. Since the pure strategy NEs of this game are $\{H_1, G_2\}$ and $\{G_1, H_2\}$, we must look at the mixed strategy equilibrium for symmetry. The condition for indifference of Player $i \in \{1, 2\}$ is:

$$u_i(H_i, \sigma_{-i}) = 2\sigma_{-i}(G) = \sigma_{-i}(H) = u_i(G_i, \sigma_{-i})$$

Which yields in symmetric equilibrium: $\sigma_1^* = \sigma_2^* = (\frac{2}{3}H, \frac{1}{3}G)$.

For the correlated equilibrium, we can use the one we derived in Exercise 3. Note that this correlated equilibrium is symmetric as $\{H_1, G_2\}$ and $\{H_1, G_2\}$ occur with the same probability. To see that the correlated NE (σ^c) dominates the mixed (σ^m), we can compute the payoffs for Player i (which is equal for both Players in each equilibrium:)

$$u_i(\sigma^m) = \sigma^m(H)\sigma^m(G) + 2\sigma^m(G)\sigma^m(H) = 2/3$$
$$u_i(\sigma^c) = \frac{1}{3}0 + \frac{1}{3}1 + \frac{1}{3}2 = 1$$

Since both players obtain a higher expected payoff in the correlated equilibrium, it Pareto-Dominates the mixed.

Exercise 5:

A strategy profile σ is trembling hand perfect if there exists a sequence of strategy profiles $\sigma^n \to \sigma$ for all i and $s_i \in S_i$ such that $\sigma_i(s_i) > 0$ implies that s_i is a best response to σ^n_{-i}. Prove that every trembling hand perfect profile is a Nash equilibrium. Give an example of a Nash equilibrium in a 2x2 game which is not trembling hand perfect and explain why.

Solution:

Let σ be a THP equilibrium. Then there is a sequence $\{\sigma^n\}$ that satisfies the conditions of the definition. Fix any player i, and fix any action $a_i \in A_i$ such that $\sigma_i(a_i) > 0$. By hypothesis:

$$u_i(a_i, \sigma^n_{-i}) \geq u_i(b_i, \sigma^n_{-i}), b_i \in A_i \quad (1)$$

Since u_i is continuous, $\sigma^n_{-i} \to \sigma_{-i}$ implies that $u_i(., \sigma^n_{-i}) \to u_i(., \sigma_{-i})$. Therefore, taking limits on each side of (1), we may conclude that

$$u_i(a_i, \sigma_{-i}) \geq u_i(b_i, \sigma_{-i}), b_i \in A_i \quad (2)$$

Notice finally that this condition applies to any action a_i with $\sigma_i(a_i) > 0$. Therefore for any two a_i and a'_i such that $\sigma_i(a_i) > 0$ and $\sigma_i(a'_i) > 0$, we must have both $u_i(a_i, \sigma_{-i}) \geq u_i(a'_i, \sigma_{-i})$ and $u_i(a_i, \sigma_{-i}) \leq u_i(a'_i, \sigma_{-i})$, so $u_i(a_i, \sigma_{-i}) = u_i(a'_i, \sigma_{-i})$. This, together with (2), implies that σ be a Nash equilibrium.

Consider the followıng game:

Table 9:

	L	R
U	$\underline{1},\underline{1}$	0,0
D	$\underline{1}$,0	$\underline{2},\underline{1}$

It is clear that $\{U, L\}$ is a NE of the above game. However it is not THP.

Take any fully mixed strategy σ_2 of Player 2. We have:

$$u_1(U, \sigma_2) = \sigma_2(L) < \sigma_2(L) + 2\sigma_2(L) = u_1(D, \sigma_2), \forall \sigma_2(R) > 0$$

Thus there are no fully mixed strategy sequences that yield best response U for Player 1. Any positive probability of Player 2 playing R makes 1 strictly prefer D.

Exercise 6

There are two groups with k each making a non-negative bid b_k. The utility of group k is:

$$u_k = (b_k - b_{-k}) - \beta \frac{(b_k - b_{-k})^2}{2} - c_k \frac{b_k^2}{2}$$

a. show that a Nash equilibrium exists and is unique
b. when is the equilibrium interior?
c. in the interior case compute the Nash equilibrium
d. how do the bids and the transfer $b_k - b_{-k}$ depend on β, c_k?
i) Becker says: higher costs lead to lower bids – is that correct?
ii) Becker says: less efficiency leads to lower transfers – is that correct?

Solution:

a) and b)

First, note that given any $c_k, \beta > 0$, u_k is obtained by subtracting a strictly convex function from a linear fnct. As a result, u_k is strictly concave in b_k. Thus, the second order condition for maximum holds. The first order condition of the maximization of u_k w.r.t. b_k (without the non-negativity constraint) is given by:

$$1 - \beta(b_k - b_{-k}) - c_k b_k = 0 \qquad (\text{FOC}_k)$$

Which can be rearranged to obtain:

$$\rho_k(b_{-k}) = b_k = \frac{1 + \beta b_{-k}}{c_k + \beta} \qquad (\text{BR}_k)$$

Similarly for group $-k$, the best response is

$$\rho_{-k}(b_k) = b_{-k} = \frac{1 + \beta b_k}{c_{-k} + \beta} \qquad (\text{BR}_{-k})$$

Which are both single valued (so indeed functions). First, note that for any $(c_k, c_{-k}, \beta) \in \mathbb{R}^2_{++}$, we have $\rho_k(b_{-k})$ strictly increasing and $\rho_k(0) > 0$. This implies that bidding zero is never a best response. Thus, under strictly positive parameters, a NE should always yield interior (strictly positive) bids.

For NE, b_k^* must be the fixed point of $\rho_k(\rho_{-k}(b_k))$. Existence of a unique NE can be seen in the explicit solution in part c.

c)

We are looking for the fixed point of $\rho_k(\rho_{-k}(b_k))$. Note that because both best response functions are linear, so is the composition. Thus it has at most one fixed point and any equilibrium is unique. Inserting (BR_{-k}) into (BR_k) yields:

$$b_k^* = \frac{1 + \beta \frac{1+\beta b_k}{c_{-k}+\beta}}{c_k + \beta}$$

Solving this for b_k, we get the unique NE bid of group k:

$$b_k^* = \frac{c_{-k} + 2\beta}{c_{-k}c_k + \beta(c_k + c_{-k})}$$

d)

Higher Costs $\Rightarrow$ Lower Bids

Differentiating b_k^* w.r.t. c_k and c_{-k}, we obtain:

$$\frac{db_k^*}{dc_k} = -\frac{(c_{-k} + 2\beta)(c_{-k} + \beta)}{(c_{-k}c_k + \beta(c_k + c_{-k}))^2} < 0, \forall (c_{-k}, \beta) \in \mathbb{R}^2_{++}$$
$$\frac{db_k^*}{dc_{-k}} = -\frac{\beta c_k + 2\beta^2}{(c_{-k}c_k + \beta(c_k + c_{-k}))^2} < 0, \forall (c_k, \beta) \in \mathbb{R}^2_{++}$$

Thus, with positive cost parameters, both bids are decreasing in both costs. The statement is true.

Less Efficiency$\Rightarrow$ Lower Transfers:

Taking β as parameter of "inefficiency", we can differentiate the transfers w.r.t. β to check this statement:

$$\frac{d(b_k^* - b_{-k}^*)}{d\beta} = -\frac{(c_{-k} - c_k)(c_k + c_{-k})}{(c_k c_{-k} + \beta(c_k + c_{-k}))^2} \lesseqgtr 0 \Leftrightarrow c_k \lesseqgtr c_{-k}$$

Which is the same condition as for $b_{-k}^* \lesseqgtr b_k^*$. Then we have that less efficiency decreases $(b_k^* - b_{-k}^*)$ when $b_{-k}^* < b_k^*$ (positive transfers) and increases it when $b_{-k}^* > b_k^*$ (negative transfers). So we have:

$$\frac{d|b_k^* - b_{-k}^*|}{d\beta} < 0$$

In absolute value of transfers. So this statement is true.