Cournot and Bertrand

The Cournot Model

- a market with *n* identical firms facing constant marginal cost *c*
- demand given by $p = a - bx$

so that the competitive solution is $(a - c)/b$ units of output and the monopoly solution is $(a - c)/2b$ units of output let $\bar{x}$ be output of representative firm

profits of a representative firm

$$\pi_i = [a - b(x_i + (n-1)\bar{x})]x_i - cx_i$$

Reaction Function

$$\frac{d\pi_i}{dx_i} = \left[a - b(2x_i + (n-1)\bar{x})\right] - c = 0$$

in a symmetric equilibrium $x_i = \bar{x}$, so

$a - b(n+1)\bar{x} = c$ giving the result

$\bar{x} = \dfrac{a-c}{b(n+1)}$ per firm

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industry output $\dfrac{n}{(n+1)} \dfrac{a-c}{b}$

when $n = 1$ this gives the usual monopoly solution

as $n \to \infty$ this approaches the competitive solution

Bertrand Competition

Firms choose prices rather than quantities

- two facing constant marginal cost *c*
- demand given by $p = a - bx$

so that the competitive solution is $(a - c)/b$ units of output and the monopoly solution is $(a - c)/2b$ units of output consumers buy from the lowest price firm: demand for firm *i*

$$x_i = \begin{cases} 0 & p_i > p_{-i} \\ \dfrac{a - p_i}{2b} & p_i = p_{-i} \\ \dfrac{a - p_i}{b} & p_i < p_{-i} \end{cases}$$

Suppose in equilibrium $p_{-i} > c$

profits are

$$\pi_i = \begin{cases} 0 & p_i > p_{-i} \\ (p_i - c)\dfrac{a - p_i}{2b} & p_i = p_{-i} \\ (p_i - c)\dfrac{a - p_i}{b} & p_i < p_{-i} \end{cases}$$

this problem does not have a solution

as $p_i \uparrow p_{-i}$ profits approach

$$(p_i - c)\frac{a - p_i}{b}$$

- always undercut by a little bit

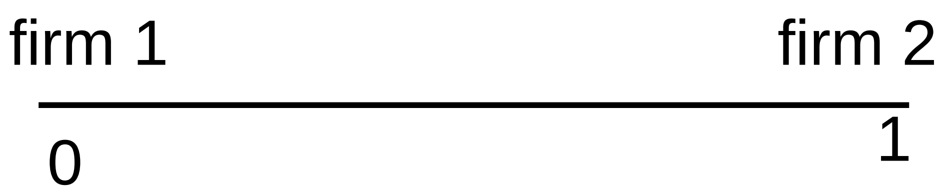
if $p_i = p_{-i} = c$ then we have a Nash equilibrium

Bertrand competition between two firms is competitive

Bertrand vs. Cournot

- choosing output is a commitment not to produce more
- in Bertrand competition firms will provide whatever amount the market wants

Bertrand Competition in the Hotelling Model



- consumers are located on the line between 0 and 1
- firms are located on each edge
- a consumer gets *b* units of satisfaction from buying 1 unit, minus *x* where *x* is the distance traveled to purchase the good, minus the price
- both firms have constant marginal cost *c*

indifference of a consumer between stores

$$b - x^* - p_1 = b - (1 - x^*) - p_2$$

solving for *x* gives demand for firm 1

firm 2 demand is 1-*x*

$$x^* = \frac{1 - p_1 + p_2}{2}$$

Reaction Function

profit $\pi_1 = (p_1 - c)\frac{1 - p_1 + p_2}{2}$

differentiate

$$\frac{d\pi_1}{dp_1} = \frac{1 - p_1 + p_2 - p_1 + c}{2} = 0$$

in symmetric equilibrium $p_1 = p_2$

$$p_1 = c + 1$$

this is valid provided

$$b - 1/2 - c - 1 \geq 0$$