Final Exam Answers: Economics 101

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1. Normal Form Games (note that a complete answer must include a drawing of the socially feasible sets)

a)

	L(*p*)	R(1-*p*)
U(*q*)	2*,5*	-1,1
D(1-*q*)	1,-1	5*,2*

Two pure strategy equilibria as marked. Mixed for player 2 $2p-(1-p)=p+5(1-p)$ so *p*=6/7; for player 1 $5q-(1-q)=q+2(1-q)$ so *q*=3/7. Pure strategy equilibria are Pareto Efficient. The mixed equilibrium has payoffs of (11/7,11/7) is not. No weakly dominated strategies. Pure strategy maxmin for both players is 1; pure strategy minmax for both players is 2.

b)

	L	R
U	5,5	-1,8*
D	8*,-1	1*,1*

Unique Nash equilibrium (U,L are both strictly dominated). No mixed equilibria due to dominance. Nash equilibrium is not Pareto efficient. Pure maxmin and maxmin is 1 for both players.

c)

	L	R
U	-1*,3	-3,5*
D	-3,5*	-1*,3

No pure strategy equilibrium. Unique Pareto efficient mixed equilibrium where both players mix 50-50. No weakly dominated strategies. Note that the socially feasible set is one-dimensional. Pure strategy maxmin for player 1 is –3, for player 2 is 3; pure strategy minmax for player 1 is –1, for player 2 is 5.

2. Long Run versus Long Run

	L	R
U	3,3	0,5
D	5,0	1,1

Use the grim strategies: U(or L) as long as UL in every past period, otherwise DR (the static Nash equilibrium). In equilibrium you get 3. If you deviate you get at most $(1-\delta)5+\delta 1 \le 3$ or $2 \le 4\delta$, so this is an equilibrium for $\delta \ge 1/2$.

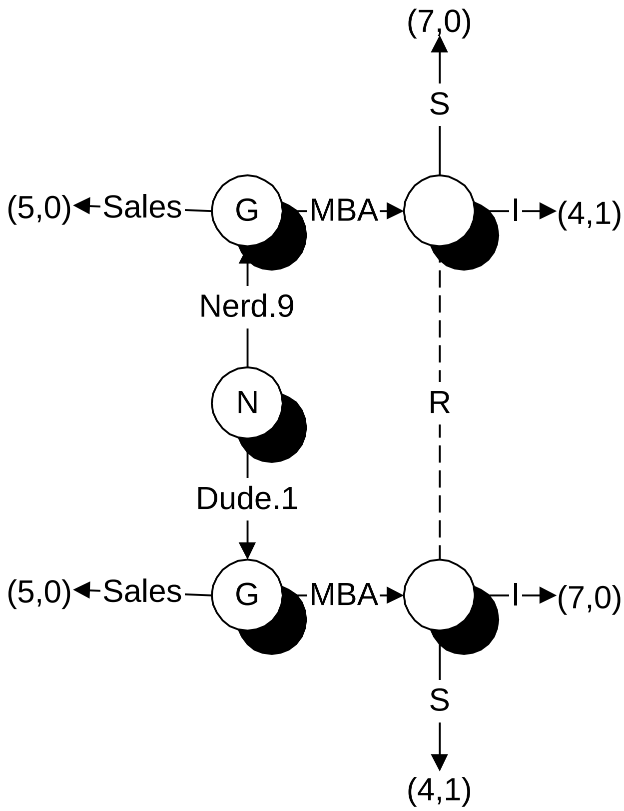
3. Long Run versus Short Run

	L	R
U	2,1*	0,0
D	11*,0	1*,3*

The unique Nash equilibrium is DR; the Stackelberg equilibrium is UL. Strategies for which lead to playing UL are UL if always UL in the past and DR if ever a deviation. Alternatively, players may base their strategies on past play of the LR player only: LR: U if U in the past and D if ever a deviation by LR and SR: L if U in the past and R if ever a deviation of the LR player.

These are optimal for the short-run player because it is in his best-response correspondence. For the long run player it must be that $2 \ge (1-\delta)11+\delta 1$ or $\delta \ge 9/10$.

4. Screening



Nerd/Dude	S(*q*)	I(1-*q*)
SS	5,0*	5,0*
SM	4.9,0.1*	5.2*,0
MS	6.8*,0	4.1,0.9*
MM	6.7,0.1	4.3,0.9*

No pure equilibria. Observe that if player 1 randomizes with weight .9 on SM and .1 on MS, he gets 5.09 regardless of how player 2 plays. So SS is strictly dominated and will not be played. Next observe that for player 2 to mix, player 1 must put some weight on SM. Suppose that 1 is indifferent between SM and MS. Then $4.9q + 5.2(1-q) = 6.8q + 4.1(1-q)$. This gives $q = 11/30$, and the expected utility is 5.09. On the other hand, the expected utility from MM is 5.18. So next we try to make player 1 indifferent between SM and MM. Then $4.9q+5.2(1-q)=6.7q+4.3(1-q)$, or $q=1/3$, with and expected utility of 5.1. In this case the utility from MS is only 5. So we conclude that $q=1/3$, and that player 1 is indifferent between SM and MM, and will not play SS or MS. Finally, to make player 2 indifferent, player 1 must choose the probability *p* of MM so that 0.9*p*=0.1, or *p*=1/9.

What then is the probability of nerd|mba?

$p(n|m)=\frac{p(m|n)p(n)}{p(m)}$ The probability of *n* is .9. The probability of *m* is equal to 1/9 (the probability of MM) plus 8/9x.1 (the probability of SM times probability of dude). The probability of m|n is 1/9, since nerds stay out when SM is played. So

$$p(n|m)=\frac{(1/9).9}{1/9+.1\times 8/9}=\frac{.9}{1+.8}=1/2$$

5. Price Discrimination

a)

$(5-p^H)x^H \geq (5-p^L)x^L$ or $5(x^H-x^L)\geq p^Hx^H-p^Lx^L$

$(3-p^L)x^L \geq (3-p^H)x^H$ or $3(x^H-x^L)\leq p^Hx^H-p^Lx^L$

Important observation: these two inequalities can be satisfied only if $x^H \geq x^L$. This in turn shows that $p^Hx^H \geq p^Lx^L$.

b)

$(5-p^H)x^H \geq 0$ or $5 \geq p^H$

$(3-p^L)x^L \geq 0$ or $3 \geq p^L$

c)

$U=.5p^Hx^H+.5p^Lx^L$

case 1: $x^H=x^L$; then from a) we see that $p^H=p^L$, so $U=p^Lx^L$. From b) we see that $p^L\leq 3$, so utility will be a maximum when $p^L=3$ and $x^L=2$, yielding a utility of 6.

Case 2: Since from a) $x^H \geq x^L$ the other case is $x^H=2$, $x^L=1$. The constraints are

$5 \geq 2p^H - p^L, 2p^H - p^L \geq 3$, $5 \geq p^H$, $3 \geq p^L$, utility is $U = p^H + .5p^L$.

Rewrite constraints $(5 + p^L)/2 \geq p^H, 2p^H - 3 \geq p^L$ so

$p^H \leq \max\{(5 + p^L)/2, 5\}$

$p^L \leq \max\{2p^H - 3, 3\}$

Case 1a) $p^H = 5$ then from second constraint $p^L = 3$, which means that $p^H \leq 4$ so this case is not possible.

Case 1b) $p^H = (5 + p^L)/2$ then $p^L \leq (5 + p^L) - 3 = p^L + 2$, which doesn't bind, so $p^L = 3$. Then $p^H = 4$, which satisfies $p^H \leq \max\{(5 + p^L)/2, 5\}$.

Utility is then $U = p^H + .5p^L = 4 + 1.5 = 5.5$

So we should sell at the fixed price of 3 and not try to price discriminate.