

## Final Exam Answers: Economics 101

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**1. Normal Form Games (note that a complete answer must include a drawing of the socially feasible sets and a description of how the answer was arrived at)**

a)

	L	R
U	5,5	0,6*
D	6*,0	1*,1*

A unique Nash equilibrium at (R,D), which is also a (strictly) dominant strategy equilibrium. Utility is (1,1) and is Pareto dominated by 5,5 so not Pareto Efficient. Pure strategy maxmin for both players is 1; pure strategy minmax for both players is 1.

b)

	L	R
U	0*,0*	-1,-100
D	-100,-1	10*,10*

Two pure strategy Nash equilibria at (U,L) and (D,R). There is also a mixed equilibrium  $-(1-p) = -100p + 10(1-p)$ , so  $p = 11/111$ . By symmetry, we get the same answer for player 2. The only efficient equilibrium is 10,10, which Pareto dominates every other outcome of the game. There are no strictly or weakly dominated strategies. Pure maxmin for both players is -1; pure minmax is 0.

c)

	L	C	R
U	1,1	2*,0	0,2*
M	0,2*	1,1	2*,0
D	2*,0	0,2*	1,1

No pure strategy equilibrium nor dominated strategies. Mixed is  $1/3, 1/3, 1/3$  (for both players, by symmetry), with an expected utility of (1,1). This is Pareto Efficient. Pure strategy equilibria are Pareto Efficient. Pure strategy maxmin for both players 0, the minmax is 2. Notice that the SFIR is empty...this is because of the use of the pure strategy rather than mixed strategy minmax (although did not need to say this to get full credit).

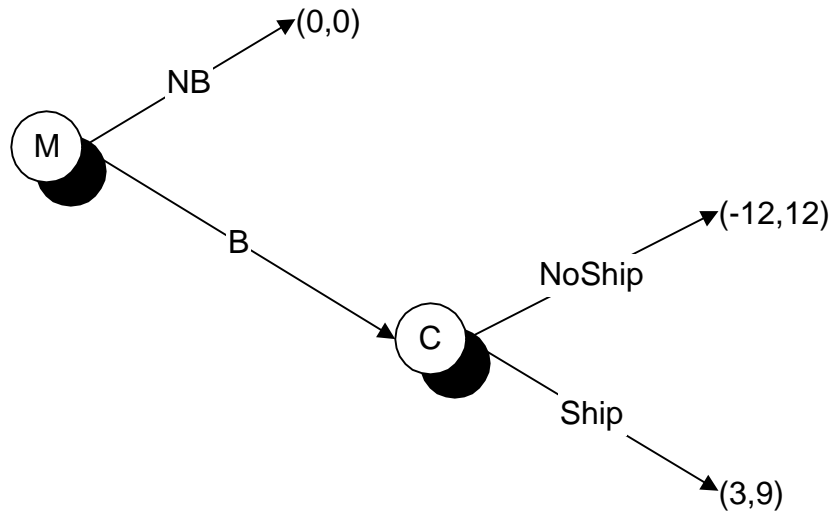
## 2. Repeated Games

	L	R
U	7,4	2,6
D	9,2	3,3

Use the grim strategies: U(or L) as long as UL in every past period, otherwise DR (the static Nash equilibrium). If player 1 deviates he gets at most  $(1-\delta)9 + \delta 3 \leq 7$  or  $2 \leq 6\delta$  or  $\delta \geq 1/3$ , If player 2 deviates he gets at most  $(1-\delta)6 + \delta 3 \leq 4$  or  $2 \leq 3\delta$  or  $\delta \geq 2/3$ . So this is an equilibrium for  $\delta \geq 2/3$ . This is subgame perfect since the punishment is a static Nash equilibrium. Minmax here is 3 for both players, so Folk Theorem says for  $\delta$  close enough to 1 that SFIR region exceeding (3,3) are all subgame perfect equilibria.

## 3. Long Run versus Short Run

M = player 1; C = player 2



	Ship	NoShip
NB	0,0*	0*,0*
B	3*,9	-12,12*

There is a unique static Nash equilibrium at (NB, NoShip) which is also subgame perfect.

The Stackelberg equilibrium is for C to precommit to Ship, resulting in a utility of 9.

Proposed repeated strategy: B and Ship as long as always B and ship in the past; otherwise NB, noShip. Obviously optimal for M since she is always playing a SR best-response. For C it must be that  $9 \geq (1 - \delta)12 + \delta 0$  or  $\delta \geq 1/4$ . Reputation isn't going to be all that useful in this example, since if the product isn't purchased, it isn't possible to establish a reputation for reliable shipments.

#### 4. Decision Analysis

no surgery = 10,001

surgery =  $.5 \times (30,001) + .5 \times 1 = 15,001$

obviously have the surgery

$$pr(+) = pr(+|like)pr(like) + pr(+|nolike)pr(nolike) = .9 \times .5 + .05 \times .5 = .475$$

so  $pr(-) = .525$

$$pr(\text{like}|+) = \frac{pr(+|\text{like})pr(\text{like})}{pr(+)} = \frac{.9 \times .5}{.475} = .95$$

$$pr(\text{like}|-) = \frac{pr(-|\text{like})pr(\text{like})}{pr(-)} = \frac{.1 \times .5}{.525} = .095$$

So surgery if + no surgery if – yields expected utility of

$.475 \times (.95 \times 30,001 + .05 \times 1) + .525 \times 10,001 = 18788$ , so you would pay \$3,788,000 for the survey

### 5. Cournot with Uncertain Cost

Consider a Cournot Duopoly with demand  $p = 17 - x$ . There are two possible levels of marginal cost: low and equal to 1 or high and equal to 3. There is a 20% chance both firms are high cost, a 20% chance they are both low cost, a 30% chance firm 1 is high cost and firm 2 low cost, and a 30% chance firm 1 is low cost and firm 2 high cost. Assuming that each firm knows its own marginal cost and these probabilities, in the Bayesian Nash equilibrium of the Cournot game, what are the equilibrium strategies of the two firms?

	1	3
1	.2	.3
3	.3	.2

Let  $x^1$  be equilibrium output of low cost firm,  $x^3$  of high cost firm

$$pr(1|1) = pr(1,1) / pr(1) = .2 / .5 = .4$$

$$pr(1|3) = pr(1,3) / pr(3) = .3 / .5 = .6$$

$$\text{Profit given low cost: } .4(16 - (x^1 + x_i))x_i + .6(16 - (x^3 + x_i))x_i = (16 - x_i - .4x^1 - .6x^3)x_i$$

$$\text{FOC is } 16 - 2x_i - .4x^1 - .6x^3 = 0 \text{ so } 16 - 2x^1 - .4x^1 - .6x^3 = 0 \text{ or } 16 = 2.4x^1 + .6x^3$$

$$\text{Profit given high cost: } .6(14 - (x^1 + x_i))x_i + .4(14 - (x^3 + x_i))x_i = (14 - x_i - .6x^1 - .4x^3)x_i$$

FOC is  $14 - 2x_i - .6x^1 - .4x^3 = 0$  so  $14 - 2x^3 - .6x^1 - .4x^3 = 0$  or  $14 = 2.4x^3 + .6x^1$  or  
 $56 = 9.6x^3 + 2.4x^1$

Subtracting these two equations give  $40 = 9x^3$  or  $x^3 = 40/9 = 4\frac{4}{9}$

Plugging back in gives  $x^1 = (16 - .6 \cdot 40/9) / 2.4 = 5.56$

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