Iterated Dominance in the Cournot Model

weak dominance never a lower payoff no matter what the opponent does, and sometimes a higher payoff

strict dominance a higher payoff no matter what the opponent does

admissibility: never use a weakly dominated strategy

If weakly dominated strategies are not used, should players anticipate that opponents will not use them?

Iterated weak dominance: eliminate weakly dominated strategies to get a smaller game, then repeat this procedure

Example of Iterated Weak Dominance

	L	M	R
U	-1,-1	2,0	1,1
M	-1,-1	1,-1	0,0
D	1,1	1,1	1,2

Eliminate M, weakly dominated by U

	L	M	R
U	-1,-1	2,0	1,1
D	1,1	1,1	1,2

Eliminate L, weakly dominated by R

	M	R
U	2,0	1,1
D	1,1	1,2

Eliminate D, weakly dominated by U

	M	R
U	2,0	1,1

Eliminate M, strictly dominated by R

	R
U	1,1

An alternative procedure

	L	M	R
U	-1,-1	2,0	1,1
D	1,1	1,1	1,2

Eliminate L **AND M**, weakly dominated by R

	R
U	1,1
D	1,2

can proceed no further

Problems with Iterated Weak Dominance

- procedure is ambiguous
- it may yield more than one answer
- it is not “robust”

Robustness

To avoid playing a weakly dominated strategy, a player must know his own payoffs exactly.

To know that his opponent is not playing a weakly dominated strategy, a player must know his opponent's payoffs exactly. This is a very strong assumption.

To know that his opponent is not playing a strictly dominated strategy, a player must only know his opponent's payoffs approximately.

A plausible (and robust) concept: iterated strict dominance, or the stronger notion of $S^{\infty}W$

Iterated Strong Dominance and Duopoly

$p = a - bx$

$a = 17, c = 1, b = 1$

so that the competitive solution is 16 units of output and the monopoly solution is 8 units of output

profits $\pi_i = [17 - (x_i + x_{-i})]x_i - x_i$

The Best Response or Reaction Function

Suppose that Macrosoft expects that Peach will produce x_{-i} units of output. What should Macrosoft do?

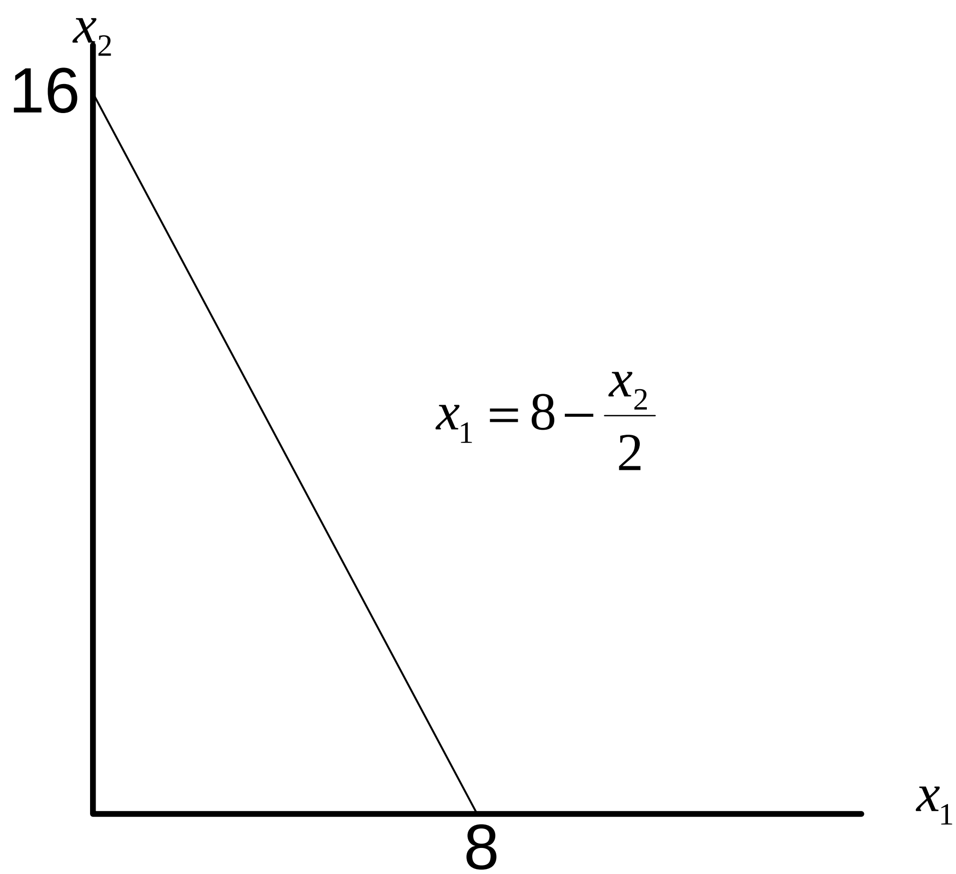
$$\pi_i = \left[17 - (x_i + x_{-i})\right]x_i - x_i$$

$$\frac{d\pi_i}{dx_i} = 16 - 2x_i - x_{-i} = 0$$

solving we find

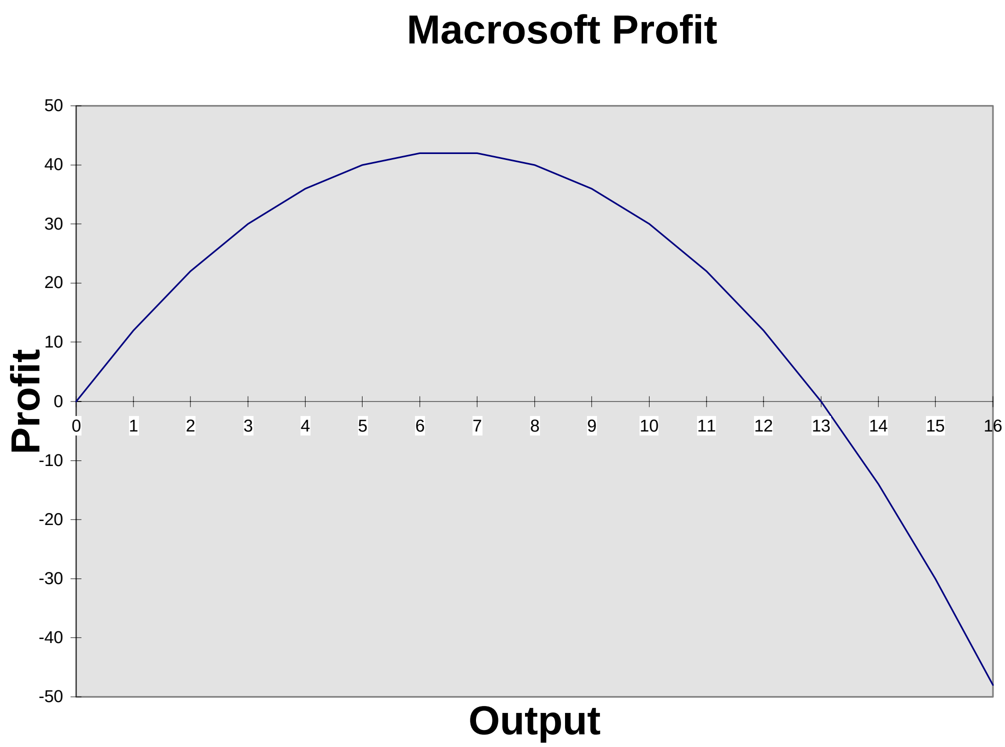
$$x_i = 8 - \frac{x_{-i}}{2}$$

This is called the *best response* or *reaction* function of Macrosoft to Peach.


x_2
16
$x_1 = 8 - \frac{x_2}{2}$
x_1
8

Profits

$$\pi_i = [17 - (x_i + x_{-i})]x_i - x_i$$



Observe that for fixed output of Peach the profit of Macrosoft is concave. We can also see this by differentiating profits twice:

$$\frac{d^2\pi_i}{dx_i^2} = -2$$

Implications of Concavity

Profits increase to the left of the optimum, and increase to the right of the optimum. Notice also from the best response function

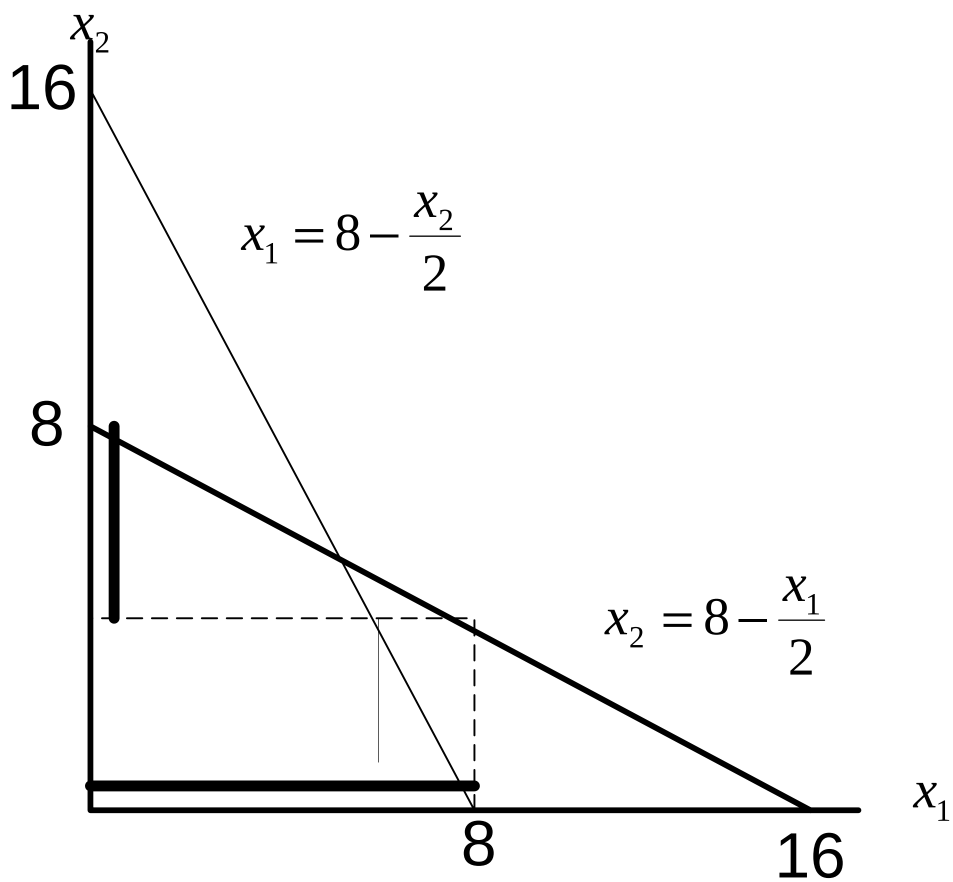
$$x_i = 8 - \frac{x_{-i}}{2}$$

that the optimum declines with output of the rival firm Peach.

- If Peach produces less than or equal to $\bar{x}$ then the best response to $\bar{x}$ strictly dominates any smaller output level
- If Peach produces greater than or equal to $\underline{x}$ then the best response to $\underline{x}$ strictly dominates any larger output level

In particular since Peach never produces less than zero, the monopoly output of 8 by Macrosoft strictly dominates any larger output level

Iterated Strict Dominance



Continuing in this way, we see that the only point that remains after iterated strict dominance is the point where the two reaction functions cross

The Cournot Equilibrium

$$x = \frac{16 - x}{2}$$

$$x = \frac{16}{3}$$

less than monopoly but more than half monopoly

industry output is twice this amount
this is 2/3 the competitive output, as against ½ for monopoly