

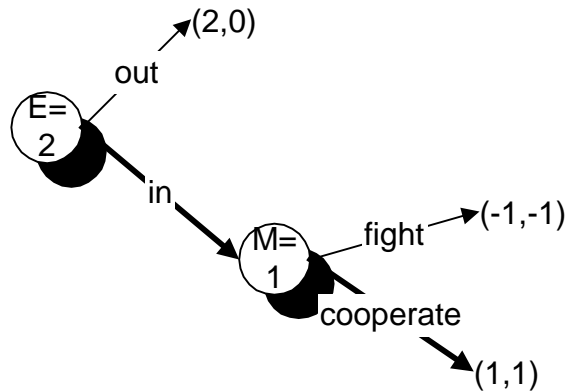
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## Answers to Problem Set 4: Dynamic Game Theory

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### 1. Long Run versus Short Run



subgame perfect equilibrium as marked

	out	in
fight	2*,0*	-1,-1
cooperate	2*,0	1*,1*

Out/fight is Nash, but isn't plausible because the incumbent wouldn't really fight.

Enter/cooperate is subgame perfect in the infinitely repeated game because it is subgame perfect in the stage game.

For the "out" equilibrium in the repeated game, note that after a failure to fight, the equilibrium is the subgame perfect "enter/cooperate" equilibrium. We must find the value of  $\delta$  for which it is actually optimal for the incumbent to fight if there is entry. (Obviously if he does so, the entrants won't wish to enter.) That is

$$(1 - \delta)(-1) + \delta 2 \geq 1$$

$$3\delta \geq 2$$

$$\delta \geq 2/3$$

Unlike the non-perfect equilibrium of the stage game, this makes sense, since the incumbent is actually willing to fight, when the penalty is entry forever afterwards when he does not.

## 2. Bayes Law

Let E be the evidence and let H be the event that the husband did it.

$$pr(H) = .8; pr(E|H) = .8; pr(E|\sim H) = .15$$

apply Bayes law

$$pr(H|E) = \frac{pr(E|H)pr(H)}{pr(E)} = \frac{pr(E|H)pr(H)}{pr(E|H)pr(H) + pr(E|\sim H)pr(\sim H)}$$

$$= \frac{.8 \times .8}{.8 \times .8 + .15 \times .20} = .96$$

so a 96% chance the husband did it. In the second case

$$pr(H|E) = \frac{.8 \times .8}{.8 \times .8 + .05 \times .20} = .98$$

## 3. Mixed Strategy Equilibrium

a) D and R are strictly dominant strategies, so this is the only Nash equilibrium.

b)

	L	R
U	3*, 2*	0, 0
D	0, 0	2*, 3*

Two pure equilibria as marked. To the symmetric mixed equilibrium let  $p$  be the probability L. Then for player 1 to be indifferent, player 2 must mix according to

$3p = 2(1 - p)$  giving  $p = 2/5$  chance of Land a  $3/5$  chance of R. For player 2 to be indifferent let  $q$  be the chance of D ; we find that  $q = 2/5$  as well.

c)

	L	R
U	4*,2	3,5*
D	2,4*	4*,2

2. No pure equilibrium. To find the mixed equilibrium, again, let  $p$  be probability of L and  $q$  be the probability of D. Then  $4p + 3(1 - p) = 2p + 4(1 - p)$  and  $4q + 2(1 - q) = 2q + 5(1 - q)$  so  $p = 1/3$  and  $q = 3/5$