Economics 201B - Final Exam

You should do three of the four questions. You have three hours. Good luck.

1. Principal-Agent

Player two must decide whether to provide an effort level of 0 or 1. Player 1 must simultaneously decide whether to pay player 2. He may choose not to pay player 2, or he may pay player 2 an amount of 4 contingent on a high effort. (Player 2 never gets paid for low effort). Player 1's utility is 0 if player 2 chooses low effort; it is $5 - w$ if player 2 chooses high effort and he pays the wage w . Player 2's utility is $\sqrt{w} - e$ where w is the wage and e the effort.

a. Find the normal form of this game.

b. Find the Nash equilibrium of this game.

c. Are there any dominated strategies?

d. Find the pure and mixed Stackelberg equilibrium in which player 1 moves first.

e. Find the minmax for both players.

Now suppose that the game is infinitely repeated

f. Player 1 is a long-run player with discount factor δ ; player 2 is a short-run player with discount factor 0. Find the set of perfect public equilibrium payoffs to the long-run player as a function of her discount factor.

g. Find strategies that support the best equilibrium from part f.

h. Player 1 and 2 are both long-run players with common discount factor δ . When δ is close to one describe the set of perfect equilibrium payoffs to both players.

2. Auto Repair

A long-lived auto repair shop with discount factor $\delta > 0$ faces a sequence of short-lived car owners. The car owners must each decide whether to have their cars repaired or not. If they do, the repair shop must decide whether to repair the car or not. If the car is not repaired, the probability it will work is $1 > \pi > 0$. If it is repaired, the probability it will work is $1 \geq \theta > \pi$. The price of the repair is $p > 0$; the cost of repair to the shop is $0 < c < p$. A car that does not work is worth nothing. A car that works is worth v. Assume that $(\theta - \pi)v > p$. Car owners can only observe whether or not the car works, not whether or not the shop repaired it. In all that follows, equilibrium means perfect public equilibrium of the infinitely repeated game with public randomization.

a. Find the extensive and normal forms of the stage-game.

b. For the long-run player, find the minmax, the static Nash, mixed precommitment and pure precommitment payoffs.

c. Find the worst equilibrium for the long-run player, and describe in general terms the set of equilibrium payoffs for the long-run player.

d. Find the best equilibrium for the repair shop as a function of the parameters.

An Auction

Consider the following auction problem: a seller has a single item for sale worth nothing to him. There are two buyers, each draws a "type" independently with equal probability from the set ($8, $12). Consider a sealed bid second price auction in which the only possible bids are $8 or $12. Let p denote the second price in the auction.

a. What is the expected revenue to the seller if each buyer i values the item equal to his own type t_i? That is, the winner gets $t_i - p$ and the loser gets 0.

b. What is the expected revenue to the seller if each buyer i gets $t_i - p$ if he wins, and $(1/2)t_{-i}$ if he loses? You may assume that type $t_i = 8$ bids $8.

Mechanism Design

There are three states of nature $\theta = (\theta_1, \theta_2) \in \{(H,H),(L,H),(H,L)\}$ each with equal 1/3rd probability. An individual has utility $u(\theta, \tilde{\theta}_1) - t(\tilde{\theta}_1, \theta_2)$ where $\tilde{\theta}_1$ is an announcement of his "type" and t is a transfer payment. Specifically let $x > 1$

$$u(H,H;L) = -x$$

$$u(H,H;H) = 0$$

$$u(L,H;L) = -1$$

$$u(L,H;H) = 0$$

$$u(H,L;L) = 0$$

$$u(H,L;H) = x$$

In addition, the only available strategies are to announce your true type, or to always announce type H. For what values of $t(\theta)$ is it a best-response for this individual to announce their true type? Is it possible to satisfy $t(H,H) = 0, t(H,L) = -t(L,H)$ and what economic interpretation does this condition have?