Normal Form Finite Games

an N player game $i = 1 \ldots N$

$P(S)$ are probability measure on S

finite strategy spaces S_i

$\sigma_i \in \Sigma_i \equiv P(S_i)$ are mixed strategies

$s \in S \equiv \times_{i=1}^N S_i$ are the strategy profiles, $\sigma \in \Sigma \equiv \times_{i=1}^N \Sigma_i$

other useful notation $s_{-i} \in S_{-i} \equiv \times_{j \neq i} S_j$

$\sigma_{-i} \in \Sigma_{-i} \equiv \times_{j \neq i} \Sigma_j$

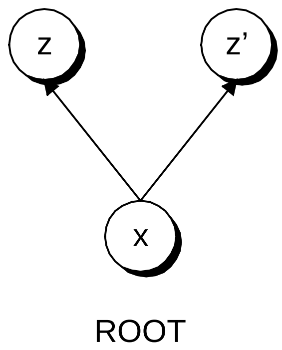
$u_i(s)$ payoff or utility; $u_i(\sigma) \equiv \sum_{s \in S} u_i(s) \prod_{j=1}^N \sigma_j(s_j)$ is expected utility

Extensive Form Finite Games

a finite game tree X with nodes $x \in X$

nodes are partially ordered and have a single root (minimal element)

terminal nodes are $z \in Z$ (maximal elements)



Players and Information Sets

player 0 is nature

information sets $h \in H$ are a partition of $X \setminus Z$

each node in an information set must have exactly the same number of immediate followers

each information set is associated with a unique player who "has the move" at that information set

$H_i \subset H$ information sets where *i* has the move

More Extensive Form Notation

information sets belonging to nature $h \in H_0$ are singletons

$A(h)$ feasible actions at $h \in H$

each action and node $a \in A(h), x \in h$ is associated with a unique node that immediately follows x on the tree

each terminal node has a payoff $r_i(z)$ for each player

by convention we designate terminal nodes in the diagram by their payoffs

Behavior Strategies

a *pure strategy* is a map from information sets to feasible actions $s_i(h_i) \in A(h_i)$

a *behavior strategy* is a map from information sets to probability distributions over feasible actions $\pi_i(h_i) \in P(A(h_i))$

Nature's move is a behavior strategy for Nature and is a fixed part of the description of the game

We may now define $u_i(\pi)$

normal form are the payoffs $u_i(s)$ derived from the game tree

Basic Concepts

- extensive versus normal form
- strategies versus behavior strategies
- Kuhn's Theorem
- weak and strong dominance
- iterated dominance and rationalizability
- best-response correspondences
- Nash equilibrium
- σ is a *Nash equilibrium* profile if for each $i \in 1,\ldots N$
  $u_i(\sigma) = \max_{\sigma'_i} u_i(\sigma'_i, \sigma_{-i})$
- existence in mixed strategies in finite games

Important Special Classes of Games

- ♦ prisoner's dilemma
- ♦ coordination games
- ♦ zero-sum games
- ♦ chain store game
- ♦ chicken
- ♦ Bayes games

Refinements of Nash Equilibrium

- trembling hand perfection
- agent normal form
- subgame perfection
- sequentiality

Extensions of Nash Equilibrium

- correlated equilibrium
- self-confirming equilibrium
- approximate equilibria

  A small portion of the population playing "non-optimally" may significantly change the incentives for other players causing a large shift in equilibrium behavior

- Quantal Response Equilibrium

Quantal Response Equilibrium

(McKelvey and Palfrey)

propensity to play a strategy

$$p_i(s_i) = \exp(\lambda_i u_i(s_i, \sigma_{-i}))$$

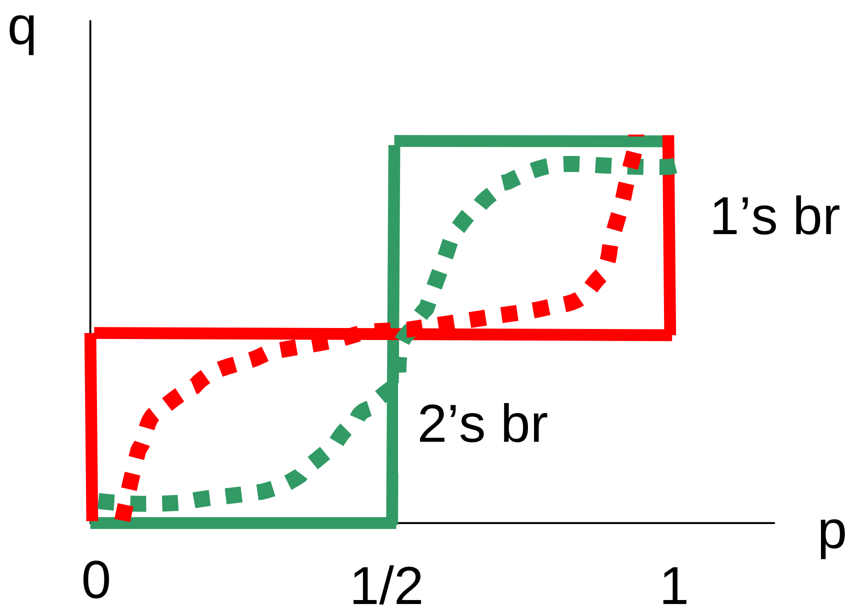
$$\sigma_i(s_i) = p_i(s_i) / \sum_{s_i'} p_i(s_i')$$

as $\lambda_i \to \infty$ approaches best response

as $\lambda_i \to 0$ approaches uniform distribution

Smoothed Best Response Correspondence Example

	L ($\sigma_2(L) = q$)	R
U ($\sigma_1(U) = p$)	1,1	0,0
D	0,0	1,1



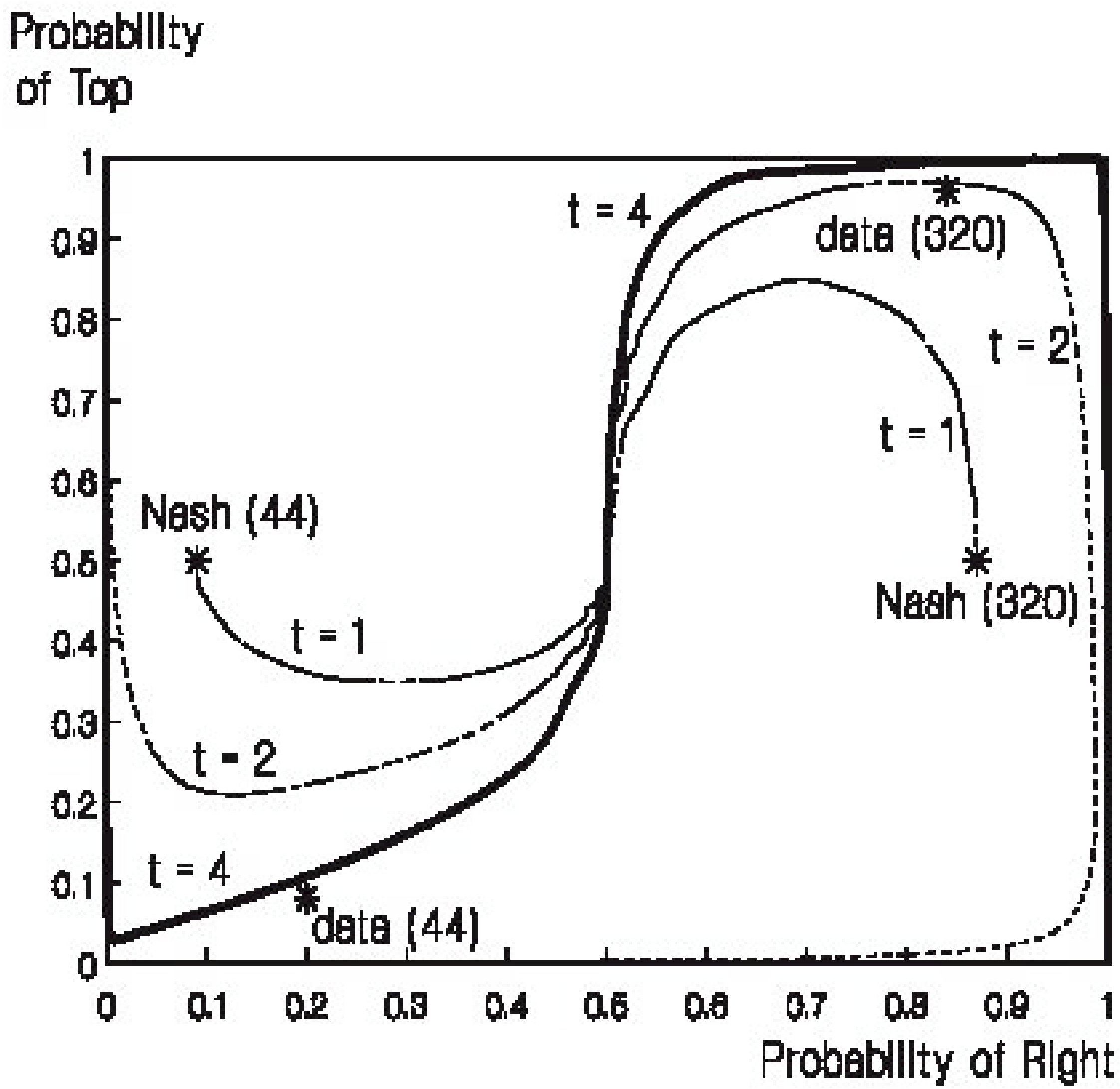
Goeree and Holt: Matching Pennies

Symmetric

	50% (48%)	50% (52%)
50% (48%)	80,40	40,80
50% (52%)	40,80	80,40

	12.5% (16%)	87.5% (84%)
50% (96%)	320,40	40,80
50% (4%)	40,80	80,40

	(80%)	(20%)
50% (8%)	44,40	40,80
50% (92%)	40,80	80,40


Probability of Top
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
t = 4
data (320)
t = 2
t = 1
Nash (44)
Nash (320)
t = 1
t = 2
t = 4
data (44)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of Right

Sequentiality

Kreps-Wilson [1982]

Subforms

Beliefs: *assessment* a_i for player *i* probability distribution over nodes at each of his information sets; *belief* for player *i* is a pair $b_i \equiv (a_i, \pi^i_{-i})$, consisting of *i*'s assessment over nodes a_i, and *i*'s expectations of opponents' strategies $\pi^i_{-i} = (\pi^i_j)_{j \neq i}$

Beliefs come from strictly positive perturbations of strategies

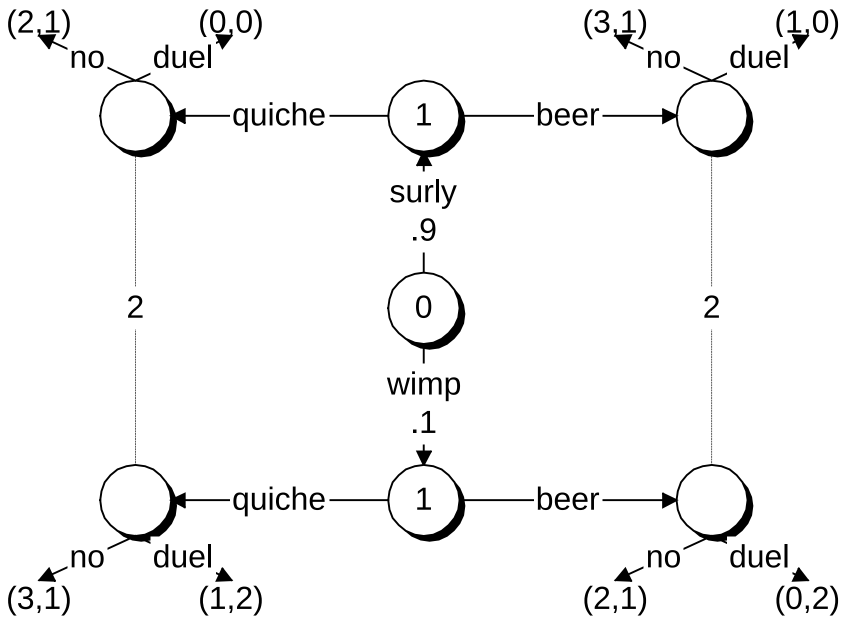
belief $b_i \equiv (a_i, \pi^i_{-i})$ is *consistent* (Kreps and Wilson [17]) if $a_i = \lim_{n \to \infty} a_i^n$ where a_i^n obtained using Bayes rule on a sequence of strictly positive strategy profiles of the opponents, $\pi^{i,m}_{-i} \to \pi_{-i}$

given beliefs we have a well-defined decision problem at each information set; can define optimality at each information set

A sequential equilibrium is a behavior strategy profile π and an assessment a_i for each player such that (a_i, π^i_{-i}) is consistent and each player optimizes at each information set

Signaling

Cho-Kreps [1987]



sequential vs. trembling hand perfect

pooling and separating

The Holdup Problem

- Chari-Jones, the pollution problem
- problem of too many small monopolies

ρ is the profit generated by an invention with a monopoly with a patent, drawn from a uniform distribution on $[0,1]$, private to the inventor

ϕ^F is the fraction of this profit that can be earned without a patent

To create the invention requires as input N other existing inventions

It costs ε / N to make copies of these other inventions, where $\varepsilon < 1/2$ and $\varepsilon / \phi^F < 1$

Case 1: Competition

if $\phi^F \rho \geq \varepsilon$ the new invention is created, probability is $1 - \varepsilon / \phi^F$.

Case 2: Patent

Each owner of the existing inventions must decide a price p_i at which to license their invention.

Subgame Perfection/Sequentiality implies that the new invention is created when $\phi\rho \geq \sum_i p_i$

Profit of preexisting owners $(1-(N-1)p - p_i)p_i$

FOC $1-(N-1)p - 2p_i = 0$

unique symmetric equilibrium $p = 1/(N+1)$

corresponding probability of invention is $1/N$