Economic 211B, David K. Levine

Answers to Problems on Repeated Games

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1. Bellman's equation

$$v_{bankrupt} = 0$$

$$v_{wealthy} = \max\begin{cases} (1-\delta)1+\delta v_{wealthy} \\ \left((1-\delta)2+\delta(pv_{bankrupt}+(1-\pi)v_{wealthy}\right) \end{cases}$$

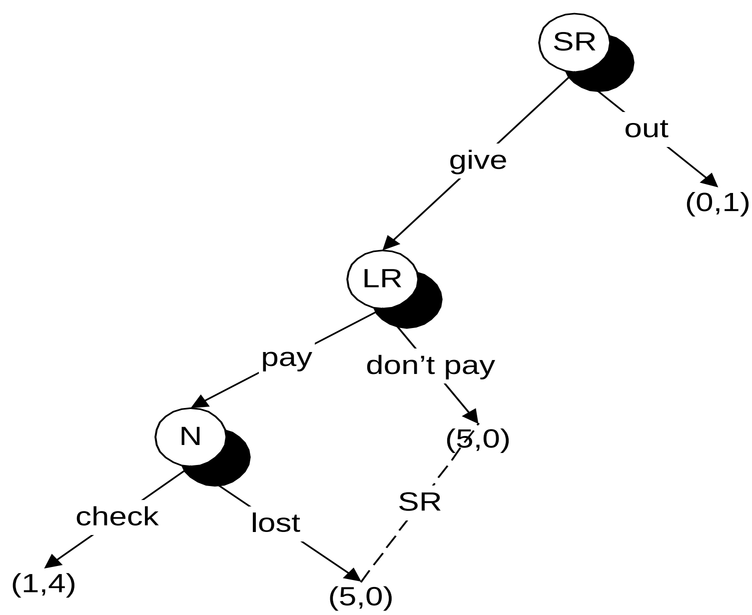
if max is bond then $v_{wealthy} = 1$

if max is stock then $v_{wealthy} = 2(1-\delta)+(1-p)\delta v_{wealthy}$

Solve second equation for $v_{wealthy}$ to find $v_{wealthy} = \dfrac{2(1-\delta)}{1-\delta(1-p)}$

Stocks better for $\dfrac{2(1-\delta)}{1-\delta(1-p)} \geq 1$ or rewrite as $1-\delta \geq \delta p$

2. a)



	give	don't
pay	3,2	0,1
don't	5,0	0,1

(b) minmax=static nash=0; maxmax=5, mixed precommitment is 50-50 yielding 4; pure precommitment is 3

(c) since minmax = static nash=0 this is also the worst equilibrium; the set of equilibrium payoffs is the line segment from 0 to $\bar{v}$

(d) best for lr is to have giving; requires at least a .5 chance of paying; if lr pays and sr gives then lr receives 3, so $\bar{v} = 3$;

also from incentive constraint $\bar{v} \geq (1-\delta)5 + \delta 0$, so $3 \geq (1-\delta)5, \delta \geq 2/5$

(e) incentive constraints

$\bar{v} = (1-\delta)3 + \delta(.5w(p) + .5w(n))$

$\bar{v} \geq (1-\delta)5 + \delta w(n)$

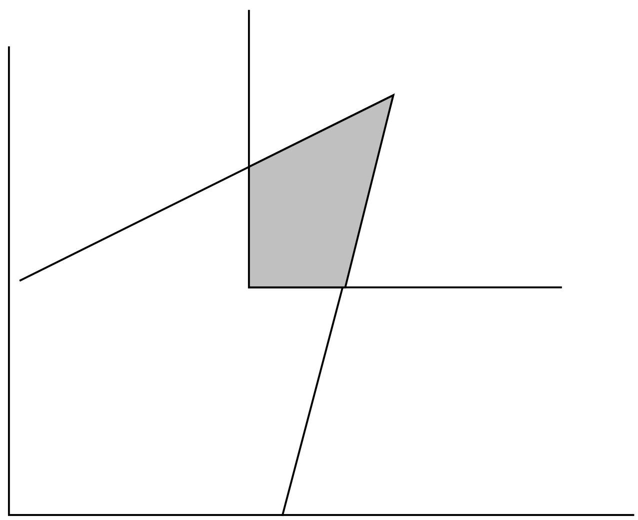
maximization of $\bar{v}$ requires that second hold with equality and that $w(p) = \bar{v}$;

solving yields $\bar{v} = 1$; $w(n) = \dfrac{1-(1-\delta)5}{\delta} \leq 1, \delta \geq 4/5$

3)

2*,2*	1,0
0,1	0,0

a) Static nash is 2,2; also the unique pareto efficient point
Minmax is 1,1
b)



c) bot for k periods, then top forever, provided no deviation; if deviation, start over again. Utility is $2\delta^k$

$2\delta^k = 1.5$

$\delta^k = 3/4$

if deviate in initial period get $(1-\delta) + 2\delta^{k+1}$.
condition for equilibrium is

$2\delta^k \geq (1-\delta) + 2\delta^{k+1}$

$0 \geq (1-\delta) + 2\delta(3/4) - 2(3/4)$

$= 1 - \delta + 3\delta/2 - 3/2 = \delta/2 - 1/2$

so this works for any δ, k combination with $\delta^k = 3/4$

d) pick δ, k as above. $\eta \in I = (0,1,2,\ldots,k)$. If you both have flag 0 play top; if either has flag $\eta > 0$ play bot. If you both have flag 0 and you play top you get flag $\max\{\eta - 1, 0\}$. If you play bot you get flag *k*. If either has flag $\eta > 0$ and you play top you get flag k; if you play bot you get flag $\max\{\eta - 1, 0\}$. Everyone starts with flag *k*.