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Economics 201B - Final Exam Answers

1. Hunter-Gatherer

Two players must decide whether to be hunters or gathers. If both are hunters, both receive 0; if both are gatherers both receive 1. If one is a hunter and one a gatherer, the hunter receives 3 and the gatherer 2.

a. Find the normal form of this game.

	H	G
H	0,0	3,2 Nash
G	2,3 Nash	1,1

b. Find the Nash equilibrium of this game. Mixed at 50-50

c. Are there any dominated strategies? No

d. Find the pure and mixed Stackelberg equilibrium in which player 1 moves first. Commit to H; mixed same as pure since 3 is the highest possible payoff

e. Find the minmax for both players. 1.5 when opponent randomizes 50-50 [most common error on this problem: finding a minmax of 2 which is only true for pure strategies]

Now suppose that the game is infinitely repeated

f. Player 1 is a long-run player with discount factor δ; player 2 is a short-run player with discount factor 0. Find the set of perfect public equilibrium payoffs to the long-run player as a function of her discount factor. Worst is static Nash at 1.5; best is static Nash at 3; true for all discount factors

g. Find strategies that support the best equilibrium from part f. play the static Nash every period

h. Player 1 and 2 are both long-run players with common discount factor δ. When δ is close to one describe the set of perfect equilibrium payoffs to both players. In other words, draw the SFIR. People generally did this right, except that many people had computed the minmax wrong.

i. Find a discount factor and strategies for part h such that both players receive an equilibrium payoff of 2.5. Public randomization between the static equilibria of (3,2) and (2,3) works for all discount factors.

2. Greenspan

A long-lived central bank faces a short-run representative consumer. The bank must decide whether or not to inflate; the consumer must decide whether or not to expect inflation. If the consumer guesses correctly, she gets 1; incorrectly she gets 0. Central bank payoffs are

	Guess inflate	**Guess not**
inflate	0	2
not	0	1

As a result of whether or not the central bank chose to inflate, economic activity is determined: there are two possibilities hyperinflation or price stability. If the bank chose to inflate the probability of hyperinflation is 1; if the bank chose not to inflate, the probability of hyperinflation is 10%. In all that follows, equilibrium means perfect public equilibrium of the infinitely repeated game with public randomization.

There were several correct ways of doing this depending on whether you interpreted the payoffs as depending on intended or actual inflation. It didn't make much difference to the answer except for a minor numerical difference in the payoffs. Here I assume that it is the choice of the central bank that determines payoffs, and the hyperinflation is just a signal of the underlying core rate.

a. Find the extensive and normal forms of the stage-game. Player 1 is central bank

0,1	2,0
0,0	1,1

b. For the long-run player, find the minmax, the static Nash, mixed precommitment and pure precommitment payoffs. Minmax is 0; static Nash is 0; pure precommitment is 1, mixed precommitment is 1.5.

c. Find the worst equilibrium for the long-run player, and describe in general terms the set of equilibrium payoffs for the long-run player. Worst equilibrium is static Nash at 0; payoffs are a line segment from 0 to some $\overline{v}$

First assume that the consumer can observe whether or not the central bank inflates.

d. Find the best equilibrium for the central bank as a function of the discount factor. For best equilibrium to be 1 must have $(1-\delta) \geq \delta$ or $\delta \geq 1/2$

Now assume that the consumer cannot observe whether or not the central bank inflates but whether or not there is hyperinflation.

e. Find the best equilibrium for the central bank as a function of the discount factor. Try to sustain low inflation:

$$\overline{v} = (1-\delta) + \delta(.9\overline{v} + .1w)$$
$$\overline{v} = (1-\delta)2 + \delta w$$

from which $\overline{v} = 8/9$, and $w \geq 0$ implies $\delta \geq 5/9$

Auctions

Consider the following auction problem: a seller has a single item for sale worth nothing to him. There are three buyers, each of whom values the item at $10. Each submits a bid to the seller. If the bid is accepted, the buyer gets the item and pays the bid (first price auction). Let m_i be the monetary payoff to player i. Suppose that player i's utility (including that of the seller) is $m_i - cm_{-i}$ where $-1 < c < 1$. You may assume that when the seller is indifferent between the buyer who receives the item he gives it to each with equal probability.

The key here is that *c* doesn't really matter. That is, the seller doesn't care who he sells to, but always prefers to sell at a higher price; as long as the item is sold, the utility the buyer receives on account of the seller's monetary payoff doesn't depend on who bought the item [the most common error appeared to be giving the buyer utility from the seller only when the buyer got the item] and the buyer always prefers to buy at a lower price, but always prefers to win the item as long as it is less than his reservation utility of 10. Note that you won't pay more than 10 regardless of *c* because |*c*|<1 and because you receive the utility -10*c* regardless of who bought the item.

a. Draw a sketch of the extensive form (no need to draw the entire thing).

b. Find a subgame perfect equilibrium for the different values of c, including $c \neq 0$.

Some one bids $10 and the item goes with equal probability to all bidders at $10.

c. How does the solution depend on c?

Mechanism Design

A risk averse consumer with utility $\log c$ has equal probability of endowment 1 or 10. A risk neutral insurance company offers a contract based on the statement of the consumer about her endowment. A consumer with a high endowment may misrepresent and pretend to have a low endowment. A consumer with a low endowment may not misrepresent. After the endowment is realized, the insurance company discovers the type (endowment) of the consumer with probability π, and if the type is observed may impose a penalty on the consumer. However, regardless of the state and the contract, the consumer may always "run away" and consume 1. In other words, if the result of the state and the contract is a level of consumption less than 1, the consumer gets 1 instead. What is the optimal contract?

This problem turned out to be pretty hard. The main thing I looked for in the answer was some facsimile of the incentive constraint $(1-\pi)\log(10+x)+\pi\log p \geq \log(10-y)$, that the high value not wish to misrepresent.