Suggested Answers for the Final Exam 2005
Course 201B

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1 Fishing

a. Normal form

	F	H
F	**1,1**	0,-8
H	-8,0	**4,4**

b. Two pure NE (F, F) and (H, H). One mixed NE: $\left[\left(\frac{4}{13}, \frac{9}{13}\right), \left(\frac{4}{13}, \frac{9}{13}\right)\right]$.

c. No.

d. Pure Stackelberg $ps^1 = 4$, mixed Stackelberg $ms^1 = 4$

e. The minmax for both players is the payoff they get in the mixed NE: $m^1 = m^2 = \frac{4}{13}$

f. (F, F) is $\frac{1}{2}-$dominant and therefore risk dominant.

g. $\underline{v}^1 = \frac{4}{13}$ and $\bar{v}^1 = 4$. All equilibria $v^1 \in \left[\frac{4}{13}, 4\right]$ can be enforced for any value of δ as long as there exists a public randomization device. The reason for this is that the lowest and highest possible payoffs are static NE.

h. Play (H, H) no matter what.

i. The goal is to find SFIR. For $\lambda_1, \lambda_2, \lambda_3 \geq 0$ and $\lambda_1 + \lambda_2 + \lambda_3 \leq 1$

$$SF = \left\{v \in R^2 : v = \lambda_1 (1, 1) + \lambda_2 (0, -8) + \lambda_3 (-8, 0) + (1 - \lambda_1 - \lambda_2 - \lambda_3) (4, 4)\right\}$$

$$IR = \left\{v \in R^2 : v \geq \left(\frac{4}{13}, \frac{4}{13}\right)\right\}$$

$$SFIR = SF \cap IR$$

A graph would be nicer, of course.

j. Supposing that the actual strategies are observable, let player 1 mix with $\Pr(F) = \frac{1}{4}$ and player 2 play H. If anybody deviates play (F, F) forever. Player

2 is playing a strict best response and does not want to deviate, hence any δ works for him. If player 1 deviates she gets 4 once and 1 forever after. Her IC is

$$3 \geq (1-\delta) 4 + \delta 1$$

$$\delta \geq \frac{1}{3}$$

satisfies the above IC.

2 Bad Reputation

a. If $p = 1$, then only the honest mechanic exists. Behavior strategies for the first consumer are $\pi^1 \in \{G, N\}$, go to the mechanic or not. Behavior strategies for the second consumer are more complicated than for the first consumer. The second consumer may have observed a report of E or T by the mechanic or nothing if the first consumer exited. Call the three information sets associated to these observations e, t, n. A behavior strategy for the second consumer is therefore, $\pi^2 \left(h^2\right) \in \{G, N\}$ for $h^2 \in H^2 = \{e, t, n\}$.

The mechanic observes whether the car needs a new engine or a tune-up if the consumer chose G. The information sets for the mechanic at the first stage are $H_1^m = \{\varepsilon_1, \tau_1\}$. ε_1 if the first car has engine trouble and τ_1 if the first car needs a tune-up. The strategies for the mechanic at the first stage are $\pi_1^m \left(h_1^m\right) \in \{E, T\}$, $h_1^m \in H_1^m$. At the second stage the mechanic's information sets depend on the signal in the first period, his report in the first period and the signal seen in the second period if the first consumer entered, and only on the signal if the first consumer didn't enter.

$$H_2^m = \{\varepsilon_1 e_1 \varepsilon_2, \varepsilon_1 e_1 \tau_2, \varepsilon_1 t_1 \varepsilon_2, \varepsilon_1 t_1 \tau_2, \tau_1 e_1 \varepsilon_2, \tau_1 e_1 \tau_2, \tau_1 t_1 \varepsilon_2, \tau_1 t_1 \tau_2, n\varepsilon_2, n\tau_2\}$$

For example, the information set $\varepsilon_1 e_1 \tau_2$ means that the first consumer entered, the signal in the first stage was engine trouble, the announcement by the mechanic was E, the second consumer entered and the signal was tune-up. The information set $n\varepsilon_2$ means that the first consumer didn't go to the mechanic but the second consumer did and the signal was engine trouble. A behavior strategy for the mechanic at the second stage is $\pi_2^m \left(h_2^m\right) \in \{E, T\}$, $h_2^m \in H_2^m$. A behavior strategy for the mechanic (at any stage) is $\pi^m \left(h^m\right) \in \{E, T\}$, $h^m \in H^m = H_1^m \cup H_2^m$.

Let's find sequential equilibria. By backward induction, the mechanic will choose to tell the truth at all the final information sets, i.e. $\pi_2^m \left(h_2^m\right) = E$ if the last element of h_2^m is ε_2 and $\pi_2^m \left(h_2^m\right) = T$ if the last element is τ_2. Given that, the second consumer will play G at all his information sets. At the previous stage the mechanic will always tell the truth again, and this will cause consumer

1 to go to the mechanic. Consequently, there is a unique sequential equilibrium which delivers a payoff of 2 to each consumer and $2+2\delta$ to the mechanic.

b. What if $p<1$? The evil mechanic is like a machine, hence his strategies don't need to be modeled. Information sets and behavior strategies are as before for all players. The only thing that changes is the amount of nodes in the information sets of the consumers, but there was no reference to them in the notation so far.

Find the values for p such that it is worthwile to go to the mechanic assuming that the honest mechanic always tells the truth. If the honest mechanic always tells the truth, then a customer who is lucky enough to get him obtains a payoff of 2. In the unfortunate event that the mechanic turns out to be evil, the expected payoff is the average of -4 and 2 (the mechanic always says E and is correct half of the time). The probability of getting the good mechanic is p. Therefore, it will be (strictly) preferable to go to the mechanic if

$$p\times 2+(1-p)\left[\frac{1}{2}\times(-4)+\frac{1}{2}\times 2\right]>0$$

This implies

$$p>\frac{1}{3}$$

The goal is to find values for $p>\frac{1}{3}$ so that in a sequential equilibrium the first consumers doesn't go to the mechanic.

By backward induction, the honest mechanic will always tell the truth at the second stage. This means that $\pi^2(t)=G$ since observing t reveals that the mechanic is honest (since he is the only type that has this action available). What is the optimal choice at information set e? Denote the second consumer's belief that she is facing the honest mechanic telling the truth at information set e by μ_e. This consumer will choose G only if[1]

$$\mu_e\times 2+(1-\mu_e)\left[\frac{1}{2}\times(-4)+\frac{1}{2}\times 2\right]\geq 0$$

which requires

$$\mu_e\geq\frac{1}{3}$$

In any sequential equilibrium, μ_e has to be consistent, which means

$$\mu_e=\frac{p\times\frac{1}{2}\times\Pr\left(G^1\right)\left[\Pr\left(E|\varepsilon_1\right)\right]}{p\times\frac{1}{2}\times\Pr\left(G^1\right)\left[\Pr\left(E|\varepsilon_1\right)+\Pr\left(E|\tau_1\right)\right]+(1-p)\Pr\left(G^1\right)}$$

[1] This calculation is conservative in that it assumes that an honest mechanic who observes τ_1 does not report E. Otherwise, the resulting condition on μ_e would be even stronger.

In this equation $\Pr\left(G^1\right)$ denotes the probability of consumer 1 playing G, $\Pr\left(E|\varepsilon_1\right)$ is the probability of the honest mechanic reporting E when he observes ε_1, and $\Pr\left(E|\tau_1\right)$ is the probability of the mechanic reporting E when he observes τ_1. Imposing the lower bound on μ_e and simplifying

$$\mu_e = \frac{\Pr\left(E|\varepsilon_1\right)}{\Pr\left(E|\varepsilon_1\right) + \Pr\left(E|\tau_1\right) + 2\frac{1-p}{p}} \geq \frac{1}{3}$$

To be conservative, consider the most favorable case for this inequality to hold, when $\Pr\left(E|\tau_1\right) = 0$

$$\mu_e = \frac{\Pr\left(E|\varepsilon_1\right)}{\Pr\left(E|\varepsilon_1\right) + 2\frac{1-p}{p}} \geq \frac{1}{3}$$

This requires that

$$3\Pr\left(E|\varepsilon_1\right) \geq \Pr\left(E|\varepsilon_1\right) + 2\frac{1-p}{p}$$

$$\Pr\left(E|\varepsilon_1\right) \geq \frac{1-p}{p}$$

Since a probability has to be less than 1

$$1 \geq \Pr\left(E|\varepsilon_1\right) \geq \frac{1-p}{p}$$

$$p \geq \frac{1}{2}$$

This implies that it is in the best interest of the second consumer to choose $\pi^2\left(e\right) = N$ when the world is populated by a fraction $p < \frac{1}{2}$ of honest mechanics.

From now on consider $p \in \left(\frac{1}{3}, \frac{1}{2}\right)$. For such a value of p, it has been shown that $\pi^2\left(t\right) = G$ and $\pi^2\left(e\right) = N$. What is the optimal choice of the honest mechanic at the first stage? If he observes ε_1 and reports truthfully he obtains 2 from the first consumer and 0 from the second consumer. If he lies and reports T he gets 0 from the first consumer and 2δ from the second consumer. Since $\delta > 1$, he chooses to report T regardless of the signal.

How does the first consumer play if she knows that the mechanic will always report T? If she goes to the mechanic, he will tell the truth half of the time and she gets $\frac{1}{2} \times 2 + \frac{1}{2} \times (-4) = -1$. This means that she will choose $\pi^1 = N$.

Finally, given that consumer 2 is at node n, what does she infer about the likelihood of facing the honest mechanic? She thinks that he will be honest with probability $p > \frac{1}{3}$. Therefore, her optimal choice is $\pi^2\left(n\right) = G$.

Summing up, for $p \in \left(\frac{1}{3}, \frac{1}{2}\right)$ behavior strategies in any sequential equilibrium are $\pi^1 = N$ (the first consumer doesn't go to the mechanic). $\pi_1^m\left(\varepsilon_1\right) = \pi_1^m\left(\tau_1\right) = T$ (the mechanic always reports tune-up in the first stage), the mechanic always tells the truth in the last stage, i.e. $\pi_2^m\left(h_2^m\right) = E$ if the last element of h_2^m is ε_2 and $\pi_2^m(h_2^m) = T$. The second consumer plays $\pi^2(t) = G$, $\pi^2(e) = N$ and $\pi^2(n) = G$.

3 Bargaining

a. After sketching the game tree it should be clear that strategies are as follows. Strategies are offers by player 1, $a^1 \in A^1 = \{1, 2, ..., 9\}$, and responses by player 2, $a^2 \in A^2 = \Pi_{j=1}^{9}\{A, R\}$. Denote as $a^2(k)$ the kth element of a^2.

b. The payoff for player 1 is

$$u^1(a^1, a^2) = \begin{cases} 10 - (1+c)\,a^1 & \text{if } a^2(a^1) = A \\ 0 & \text{if } a^2(a^1) = R \end{cases}$$

Player 2 will accept only if

$$a^1 - c\left(10 - a^1\right) \geq 0$$

$$a^1 \geq \frac{10c}{1+c} = \frac{10}{1+\frac{1}{c}}$$

By backward induction, player 1 has to take into account the response by player 2. (Assume that player 2 accepts when indifferent)

$$u^1\left(a^1, BR^2(a^1)\right) = \begin{cases} 10 - (1+c)\,a^1 & \text{if } a^1 \geq \frac{10c}{1+c} \\ 0 & \text{if } a^1 < \frac{10c}{1+c} \end{cases}$$

The problem for player 1 becomes

$$\max_{a^1 \in A^1} 10 - (1+c)\,a^1$$

subject to

$$a^1 \geq \frac{10c}{1+c}$$

The solution to this problem is choosing the smallest number $a^1 \in A^1$ that satisfies the constraint. For a fixed c call that number a_c^{1*}. Then the SGPNE strategies are $\left(a_c^{1*}, a_c^{2*}\right)$, where a_c^{2*} is such that $a^2(k) = R$ for all $k < a_c^{1*}$ and $a^2(k) = A$ for all $k \geq a_c^{1*}$.

c. The solutions are indexed by c. Note that $a^1 = 5$ satisfies the constraint for values of c arbitrarily close to 1. If $c = 0$ then $a^1 = 1$ is the optimal choice for player 1. There will be cut-off points that determine what the solution looks like in between. The cut-offs can be calculated by using the constraint with equality for $k = 0, 1, 2, 3, 4, 5$

$$k = \frac{10c_k}{1+c_k}$$

Solving for c_k

$$c_k = \frac{k}{10-k}$$

Thus, $c_0 = 0, c_1 = \frac{1}{9}, c_2 = \frac{1}{4}, c_3 = \frac{3}{7}, c_4 = \frac{2}{3}, c_5 = 1$. The general rule for choosing a_c^{1*} is

$$\text{if } c \in [c_{k-1}, c_k] \text{ then } a_c^{1*} = k \text{ is a solution}, \qquad k = 1,2,3,4,5$$

a_c^{2*} is constructed from a_c^{1*} as described in part b.

4 Risk Aversion

a. Use an equation of the form $u(x-p) = Eu(x+\sigma y)$ with $E[y] = 0$ and $E\left[y^2\right] = 1$ to derive a formula for calculating p using a Taylor Series Expansion.

Start with

$$u(x-p) = Eu(x+\sigma y)$$

Taylor Series expansion around x

$$u(x) - pu'(x) = E\left[u(x) + \sigma y u'(x) + \frac{1}{2}\sigma^2 y^2 u''(x)\right]$$

Distribute the expectation

$$u(x) - pu'(x) = u(x) + \sigma u'(x) E[y] + \frac{1}{2}\sigma^2 u''(x) E\left[y^2\right]$$

Use $E[y] = 0$ and $E\left[y^2\right] = 1$ and solve for p

$$p = -\frac{u''(x)}{u'(x)}\frac{\sigma^2}{2}$$

b. Assume a CES utility function. Use the expression you derived in part a. to get an expression relating the coefficient of relative risk aversion to wealth.

The CES function with parameter relative risk aversion parameter γ

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

Therefore, $u'(x) = x^{-\gamma}$ and $u''(x) = -\gamma x^{-\gamma-1}$

In the expression from a.

$$p = -\frac{-\gamma x^{-\gamma-1}}{x^{-\gamma}}\frac{\sigma^2}{2} = \frac{\gamma}{x}\frac{\sigma^2}{2}$$

Now calculate p and σ^2 from the data.

Use $E[y]=0$ to calculate the implied value for p. The expected payoff of the gamble is

$$\frac{1}{3}\times\left[\frac{39}{4}-\frac{12}{4}-\frac{9}{4}\right]=\frac{6}{4}=\frac{3}{2}=p$$

The variance $\sigma^2=\frac{1}{3}\times\left[\left(\frac{39}{4}-\frac{6}{4}\right)^2+\left(-\frac{12}{4}-\frac{6}{4}\right)^2+\left(-\frac{9}{4}-\frac{6}{4}\right)^2\right]=\frac{1}{48}\left[33^2+18^2+15^2\right]=\frac{1089+324+225}{48}=\frac{273}{8}$, and $\sigma=\sqrt{\frac{273}{8}}$

Hence, $u(x-p)=Eu(x+\sigma y)$ can be written as

$$u\left(x-\frac{3}{2}\right)=Eu\left(x+\sqrt{\frac{273}{8}}y\right)$$

Therefore, $p=\frac{\gamma}{x}\frac{\sigma^2}{2}$ becomes

$$\frac{3}{2}=\frac{\gamma}{x}\frac{273}{16}$$

$$x=\gamma\frac{273}{16}\times\frac{2}{3}=\gamma\frac{91}{8}$$

The expression relating the coefficient of relative risk aversion to wealth is linear

$$x=\frac{91}{8}\gamma$$

c. If wealth is \$350,000, what is the coefficient of relative risk aversion?

Use the formula to solve for γ

$$\gamma=\frac{8}{91}\times 350000=30769$$

d. If the coefficient of relative risk aversion is 20, what is wealth?

Use the formula

$$x=\frac{91}{8}\times 20=227.5$$

e. If preferences are logarithmic, what is wealth?

Logarithmic preferences correspond to $\gamma=1$

$$x=\frac{91}{8}=11.375$$

f. For what measure of wealth does the answer in part e. make sense?

If wealth means money in your pocket.