Economics 201B: Decision Theory Problem Set (#3)

Elicitation of Beliefs

An experimental subject believes the probabilities over the finite state space Ω are μ. He is asked to report these probabilities. In an effort to get the subject to report truthfully, the experimenter offers a reward schedule $r(\omega, \tilde{\mu})$ where ω is the realized state of nature, and $\tilde{\mu}$ is the reported probability distribution. Suppose the reward schedule is $r(\omega, \tilde{\mu}) = \tilde{\mu}(\omega)$. What is the optimal report for the subject to make? What if the reward schedule is $r(\omega, \tilde{\mu}) = \log \tilde{\mu}(\omega)$. Is there a connection here with econometric theory?

Risk Aversion in the Lab

From experimental data of Peter Boessarts and Charles Plott, individuals in the laboratory are indifferent between getting nothing, and a gamble paying \$9.75, –\$3.00, –\$2.25 each with probability 1/3. For an individual with CES preferences, find the coefficient of relative risk aversion as a function of wealth, using the approximation

$$p = -\frac{u''(x)}{u'(x)}\frac{\sigma^2}{2}.$$

- If wealth is \$350,000, what is the coefficient of relative risk aversion?
- If the coefficient of relative risk aversion is 20, what is wealth?

AK model

There is a single unit of capital, and an infinite number of time periods $t = 1,2,\ldots$. Utility of the representative consumer is discounted additively separable with discount factor δ and coefficient of relative risk aversion ρ. Capital may be used either to produce consumption or to produce capital for next period. A unit of capital used to produce capital for next period produces $\beta > 1$ units of next period capital.

- Find the optimal steady state growth rate of capital and consumption.
- Find the initial value of the capital stock.
- What happens to the initial value of the capital stock as β increases?
- Interpret your result in terms of the stock market reaction to unanticipated good news.

Investment

An investor may either be wealthy or bankrupt. If he is bankrupt he receives zero and has no choices. If he is wealthy he may choose to invest in stocks or bonds. If he invests in bonds, he remains wealth, but receives a utility of only one. If he invests in stocks, he has a p chance of going bankrupt each period but receives a utility of two. For what values of p, δ should the investor buy stocks?

Welfare

A continuum of consumers has utility function $u(x) = 78x - x^2$. Each consumer has a 50% chance of getting $x = 30$ and a 50% chance of $x = 10$. Consider the following "mechanism:" a consumer that announces he has $x = 30$ pays τ. A consumer that announces $x = 10$ receives a lottery with a 50% chance of winning g and a 50% chance of winning b where $.5g + .5b = \tau$. Suppose that "rich" consumers ($x = 30$) can lie and say that they are poor $(x = 10)$, but not vice versa. What values of τ, g, b maximize the expected utility of a consumer before he knows his type, subject to the constraint that the rich consumer does not wish to lie.