Quality Ladders, Competition and Endogenous Growth

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The Standard

“Schumpeterian Competition”

“Monopolistic Competition”

innovation modeled as endogenous rate of movement up a quality ladder

incentive to innovate comes from short-run monopoly at each rung of the ladder

Romer, Aghion-Howitt, Grossman-Helpman

The Questions

Does imperfect competition have anything to do with this?

Does fixed cost of innovation have anything to do with this?

Do models where the incentive to innovate are a short-term monopoly have a better claim to fit the data well?

Benchmark Environment: Grossman&Helpman

d_j the consumption (demand) for goods of quality j

ρ be the subjective interest rate

$\lambda > 1$ a constant = increase in quality each step up quality ladder

consumer utility

$$U = \int_0^\infty e^{-\rho t} \log\left[\sum_j \lambda^j d_{jt}\right] dt$$

One unit of output requires a unit of labor to obtain

The first to reach j has monopoly until $j+1$ is reached

R&D intensity is $\tilde{\iota}$, probability of innovating is $\tilde{\iota}dt$ at a cost of $\tilde{\iota}a_I dt$.

One unit of labor, E steady state expenditure

Wage rate is numeraire and price is λ

The resource constraint is $a_I \iota + E / \lambda = 1$

Monopolist gets a share $(1-1/\lambda)$ of expenditure

Cost of innovation is a_I, rate of return is $(1-1/\lambda)E\ /a_I$.

There is a chance ι of losing the monopoly, reducing the rate of return to the interest rate

$$\frac{(1-1/\lambda)E}{a_I}-\iota=\rho$$

This and the resource constraint solve for R&D intensity

$$\iota=\frac{(1-1/\lambda)}{a_I}-\frac{\rho}{\lambda}.$$

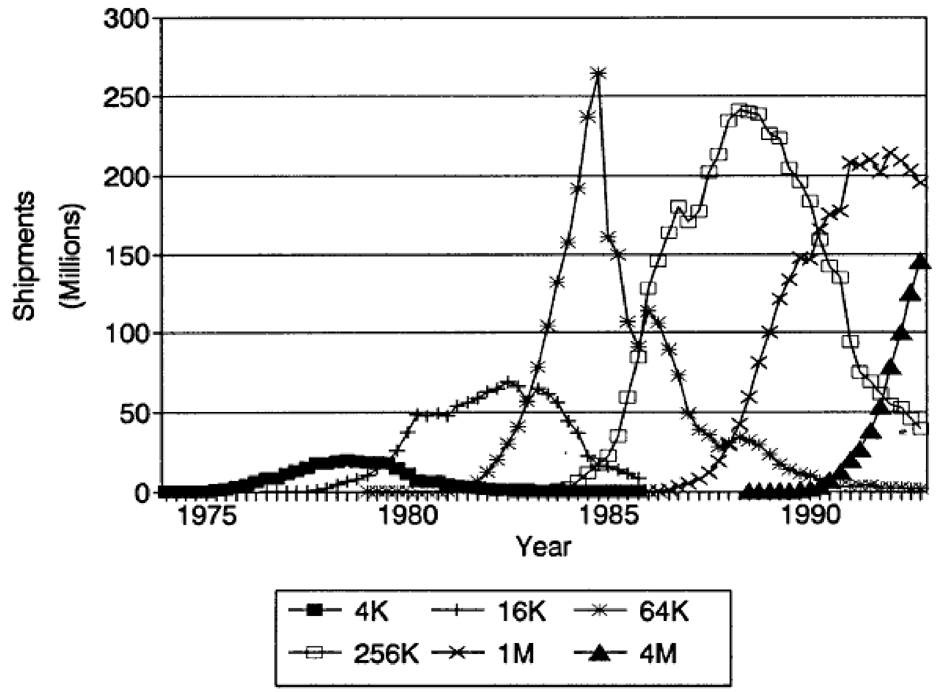
The Story

moving up the capital ladder is unambiguously good

the limitation on the rate at which you move up the ladder is the increasing marginal cost of labor used for innovation

here the increasing marginal cost of labor is because it is drawn out of the production of output – this is a trick to keep the model stationary

How industries walk up a quality ladder



Key Feature

gradual switching from one technology to the next

suggests that there is a trade-off between increasing use (ramping up) an old technology and introducing a new one

the fact suggest an alternative theory of why there is gradual movement up the quality ladder

introduce a new technology when the benefits of the old one are exhausted

quite different than the Romer, AH, GH story

in their setup the new technology is always better, it is just costly to go there right away

Innovation with Knowledge Capital

Retain preferences and ladder structure

Ladder corresponds to qualities of knowledge capital k_j and consumption d_j

One unit of labor

Consumption needs labor and knowledge capital

Knowledge capital can be used to produce consumption, more knowledge of the same quality (widening, imitation), new knowledge (deepening, innovation)

Widening, imitation: use knowledge capital to produce knowledge capital of the same quality level at rate $b > \rho$

Deepening, innovation: use knowledge capital h_j to produce knowledge capital of the next high quality level: one unit of quality $j+1$ needs $a > 1$ units of quality j

Deepening is costlier than widening, $\lambda / a < 1$.

Law of motion:

$$\dot{k}_j = b(k_j - d_j) - h_j + \frac{h_{j-1}}{a}.$$

Allow $\Delta k_{j+1} = -\Delta k_j / a$

This is an ordinary diminishing return economy, CE is efficient

Proposition: production uses at most two adjacent qualities of capital $j-1, j$

Proposition: after some initial period labor is fully employed:

$$d_j + d_{j+1} = 1.$$

Proposition: Consumption grows at a rate $b - \rho$ or not at all

Proposition: You innovate only when necessary, that is when $d_j = 1$

Proposition: Equilibrium paths cycle between widening and deepening

Widening

At the beginning of this phase $d_j = 1$ and $d_{j+1} = 0$.

Consumption during widening is $\lambda^j d_j + \lambda^{j+1} d_{j+1}$

It increases as labor shifts from old to new capital

Since $d_j(0) = 1$ and $c(0) = \lambda^j$

$$\lambda^j d_j(t) + \lambda^{j+1} d_{j+1}(t) = \lambda^j e^{(b-\rho)t}.$$

Using full employment condition

$$d_j(t) = \frac{e^{(b-\rho)t} - \lambda}{1-\lambda}.$$

This continues until $d_j(\tau_1) = 0$ and $d_{j+1}(\tau_1) = 1$

At which point widening ends.

Solving $d_j(\tau_1) = 0$ we find the length of widening

$$\tau_1 = \frac{\log \lambda}{b - \rho}.$$

Deepening

Step back at the end of the previous widening phase, when only old capital was used to produce consumption.

How much capital of new quality should we pile up before starting the new widening phase?

Until we do so, full employment implies that consumption is constant

$d_j(t) = 1,$

A reduction in consumption now give $e^{b\tau_0} / a$ units of $j+1$ capital by the end deepening.

This future capital gives consumption worth $e^{-\rho\tau_0}$ units of current consumption.

Consumption of $j+1$ quality is worth λ time consumption of quality j

At the social optimum, this shift must be neutral,

$$1 = \lambda e^{-\rho\tau_0}(e^{b\tau_0} / a).$$

$$\tau_0 = \frac{\log a - \log \lambda}{b - \rho},$$

The same flow of consumption service can be obtained through a smooth innovation process

Solve

$$\mu \int_0^{\tau_0} e^{b(\tau_0 - t)} dt = e^{b\tau_0}$$

for μ , to get $\mu = b/(1 - e^{-b\tau_0})$.

Hence there is a continuum of payoff equivalent equilibria.

Focus on the one in which innovation is done at end of deepening

Intensity of innovation

This is just the inverse of the length of the cycle, i.e. of the sum $\tau_0 + \tau_1$ of the two parts of the cycle

$$j* = \frac{b - \rho}{\log a}.$$

Length of cycle is endogenous but does not depend on how high the step ladder is

Remarks

New knowledge is costlier than old

New knowledge loses the productive capacity of the old

Conversion is instantaneous – use $ae^{b\Delta}$ time delay is capitalized into the cost of conversion

Evolution of the stocks

***Deepening*:**

growth rate of consumption is zero

value at $t = \tau_0$ of old capital converted to new is F

conversion takes place at $t = \tau_0$, hence $k_j(\tau_0) = 1 + F$

so $k_j(t) = 1 + Fe^{-b(\tau_0 - t)}$ and $d_{j+1}(t) = 0$ during deepening

Widening:

$k_j(t)$ and $d_j(t)$ shrink from 1 at $t = \tau_0$, to 0 at $t = \tau_0 + \tau_1$.

from $\dot{d}_j = -\dot{d}_{j+1}$ and $\dot{c}/c = b - \rho$ derive

$$\dot{d}_j = \frac{(b-\rho)\lambda}{1-\lambda} + d_j(b-\rho),$$

which has the solution given earlier, i.e.

$$d_j(t) = \frac{e^{(b-\rho)(t-\tau_0)}}{1-\lambda} - \frac{\lambda}{1-\lambda}$$

New capital producing consumption expands as

$$d_{j+1}(t) = \frac{1}{1-\lambda} - \frac{e^{(b-\rho)(t-\tau_0)}}{1-\lambda},$$

Plugging the results in the law of motion for $k_{j+1}(t)$, the expanding stock, we have

$$\dot{k}_{j+1}(t) = bk_{j+1}(t) - \frac{b}{1-\lambda} + \frac{e^{(b-\rho)(t-\tau_0)}}{a(1-\lambda)}\left[b(a-1)+\rho\right]$$

for $t \in (\tau_0, \tau_0 + \tau_1]$.

Solving this we find that

$$k_{j+1}(t) = -\frac{b(a-1)+\rho}{a\rho}\frac{e^{(b-\rho)(t-\tau_0)}}{1-\lambda} + \frac{1}{1-\lambda} + C$$

The initial condition $k_{j+1}(\tau_0) = (F/a)$ can be used to eliminate the constant of integration to get

$$k_{j+1}(t) = \frac{b(a-1)+\rho}{a\rho(1-\lambda)}[1-e^{(b-\rho)(t-\tau_0)}] + \frac{F}{a}.$$

For the cycle to repeat itself, at the end of the widening period the stock of capital of quality $j+1$ must equal $1+Fe^{-b\tau_0}$ again. Use this to compute F^*, the (pseudo) fixed cost invested in innovation along the competitive equilibrium path

Comparison of the Models

Ignore technical differences, stick to substantive

First, the parameter λ – two offsetting effects during widening and deepening respectively

Second, our model has the extra widening parameter b. In a certain sense the Grossman-Helpman model assumes that $b = \infty$.

Third, our model does not require a fixed cost to innovate.

Move on to this issue

Fixed Cost

Assume that there is a technologically determined fixed cost F that gets you $\bar{k} = F / a$ units of new capital.

Once the fixed cost is incurred, it is possible to convert additional units of old capital to new capital at the same rate a.

If j is introduced for the first time at t_j then $j+1$ cannot also be introduced at time t_j, hence the distance in time between t_j and t_{j+1} is either constant or infinity.

We are interested in

$F \leq F^*$ *small fixed cost*, and

$F > F^*$ *large fixed cost*.

Behavioral economics: who innovates?

Competitive means no one has monopoly power.

Can someone affect prices by innovating?

Can he/she take this into account when choosing action?

When everyone believes that nothing affects equilibrium prices

competitive equilibrium with non-atomic innovators,

When someone feels powerful, we have its “schumpeterian” perturbation:

entrepreneurial competitive equilibrium

Fixed Cost: Non- Atomic Innovators

viable initial stocks of knowledge capital

$$\vec{k}_J = (k_0, k_1, \ldots, k_J)$$

$$k_J \geq \overline{k}$$

feasible paths

$$k(t) = [k_0(t), k_1(t), \ldots, k_j(t), \ldots],\ t \in [0, \infty)$$

$$\Delta k(t) = [\Delta k_{J+1}(t_{J+1}), \Delta k_{J+2}(t_{J+2}), \ldots, \Delta k_{J+n}(t_{J+n}), \ldots]$$

Definition 1. A *competitive equilibrium with atomic innovators* E with respect to a viable $\vec{k}_J$ consists of:

(i) a non-decreasing sequence of times $(t_0, t_1, \ldots)$, with $t_j = 0$ for $j \leq J$ and, for $j > J$, either $t_j > t_{j-1}$, or $t_j = \infty$;

(ii) a path of capital $k_j(t) \geq 0$ and capital prices $q_j(t) \geq 0$ for $t \geq t_j$, and a path of consumption $c(t) \geq 0$ and consumption prices $p(t) \geq 0$ that satisfy the conditions

(1) **[Consumer Optimality]** $c(t)$ max

$$\int_0^\infty e^{-\rho t} \log(\tilde{c}(t))dt$$ subject to

$$\int_0^\infty e^{-\rho t} p(t)\tilde{c}(t)dt \leq \int_0^\infty e^{-\rho t} p(t)c(t)dt.$$

(2) [**Optimal Production Plans** at t_j]

$$\Delta k_j(t_j) = -\Delta k_{j-1}(t_j)/a \geq \overline{k}$$

$$q_j(t_j) = aq_{j-1}(t_j)$$

(3) **[Optimal Production Plans for** $t > t_j$]

$$\dot{k}_j(t) = b(k_j(t) - d_j(t)) - h_j(t) + \frac{h_{j-1}(t)}{a},$$

$$k_j(t) \geq \max\{d_j(t), h_j(t)\},$$

$$q_j(t) \leq aq_{j-1}(t) \text{ and } q_j(t) = aq_{j-1}(t) \text{ if } h_{j-1}(t) > 0,$$

$k_j(t), d_j(t)$ maximize profits

(4) [**Social Feasibility**]

$$c(t) = \sum_j \lambda^j d_{jt}(t)$$

(5) [**Boundedness**] for some number $K > 0$

$\sum_{j=0}^{\infty} k_j(t) < K$,b at all t.

Implications of the zero profit condition

$$\frac{c(\tau_0)}{c(0)} = \frac{e^{(b-\rho)\tau_0}\lambda}{a}.$$

$$(k, \tau_0, F)$$

is a candidate for equilibrium if zero profits and $ke^{b\tau_0} \geq F$ hold. Also

$$c(t) = c(\tau_0)e^{(b-\rho)(t-\tau_0)}.$$

When $t = \tau_0 + \tau_1$ $c(t) = \lambda^{j+1}$. This gives

$$c(\tau_0) = \lambda^{j+1}e^{-(b-\rho)\tau_1},$$

and, because $c(0) = \lambda^j$, the zero profit condition simplifies to

$$e^{(b-\rho)(\tau_0+\tau_1)} = a.$$

The Case of Small Fixed Cost

Theorem 1: *In the economy with a small fixed cost, for given initial conditions, there exists a unique competitive equilibrium with non-atomic innovators. This equilibrium is efficient.*

The Case of Large Fixed Cost

Assume now that $F > F^*$
The dates innovations take place less easy to pin down.

First, the competitive equilibrium of the economy without fixed cost is no longer feasible.

Second, the new competitive equilibrium is not efficient.

Equilibrium exists; in fact: quite a few equilibria exist

The set of equilibria is parameterized by $(\Delta k_j, \tau_{j,0})$

Theorem 2:

- The earliest competitive equilibrium with non-atomic innovators pareto dominates all other steady state competitive equilibria with non-atomic innovators, but is not first best.

- Given $\Delta k_j \geq F / a$, there is at most onestationary equilibrium and at least one cyclical equilibrium of period two.

- There are also equilibria with $\Delta k_j = 0$ for $j > J^*$, and $\tau_{J^*+1,0} = +\infty$

Fixed Cost: Entrepreneurial Innovators

Fix one competitive equilibrium $\hat{E}$.

A j *-innovation* is a pair $(\tilde{t}, \tilde{k}_j(\tilde{t}))$ composed of the time $\hat{t}_{j-1} < \tilde{t} < \hat{t}_j$ at which *a single agent* purchases $\tilde{F}$ units of capital of quality $j-1$ and turns them into $\tilde{F}/a$ units of capital of quality j.

We say that a competitive equilibrium $\tilde{E}$ is *a feasible continuation* for the j -innovation $(\tilde{t}, \tilde{F})$ if

(a) $\tilde{k}_{j-1}(\tilde{t}) = \hat{k}_{j-1}(\tilde{t}) - \tilde{F}$, $\tilde{k}_{j-1}(t) = \hat{k}_{j-1}(t'), \text{ for } t < \tilde{t};$

(b) $\tilde{k}_{j'}(t) = \hat{k}_{j'}(t)$ for $j' < j - 1$ and all t.

It is *Markov* if $\tilde{k}(\tilde{t}) = \hat{k}(\hat{t})$ and $\Delta\tilde{k}(\tilde{t}_{j+1}) = \Delta\hat{k}(\hat{t}_{j+1})$.

We say that a j-innovation $(\tilde{t}, \tilde{F})$ is *profitable* with respect to a feasible continuation $\tilde{E}$ if

$\tilde{q}_j(t_j) \geq a\tilde{q}_{j-1}(t_j)$, and

$\tilde{q}_j(t_j) > a\hat{q}_{j-1}(t_j)$.

Definition 2. An *entrepreneurial competitive equilibrium* is defined as a competitive equilibrium with non-atomic innovators that

(1) does not admit innovations that are profitable with respect to feasible Markov continuations,

(2) does not stop innovating, that is, $t_j < \infty$ for all j.

Theorem 3: There is a unique entrepreneurial competitive equilibrium: it is the earliest competitive equilibrium with non-atomic innovators.