Decision Theory: Time

Impatience

infinite discounted utility

$\sum_{t=1}^{\infty} \delta^{t-1} u_t$

average discounted utility

$(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} u_t$

note that average present value of 1 unit of utility per period is 1

The real equity premium puzzle

Utility $u(x) = \frac{x^{1-\rho}}{1-\rho}$, $\sum_{t=1}^{\infty} \delta^{t-1} u_t$

Consumption grows at a constant rate $x_t = \gamma^t$

$u'(x) = x^{-\rho}$

marginal rate of substitution $\frac{1}{1+r} = \frac{\delta u'(x_{t+1})}{u'(x_t)} = \frac{\delta \gamma^{-\rho(t+1)}}{\gamma^{-\rho t}} = \delta \gamma^{-\rho}$

1889-1984 from Shiller [1989]

average real US per capita consumption growth rate 1.8%

$\rho = 8.84 \;\; r = 17\%$

Mean real return on bonds 1.9%; Mean real return on S&P 7.5%

http://www.dklevine.com/econ201/interest.xls

How does the market react to good news?

Value of claims to future consumption relative to current consumption

$x_1 = 1$

$$\frac{\sum_{t=2}^{\infty} \delta^{t-1} u'(x_t) x_t}{u'(1)}$$

$$\sum_{t=2}^{\infty} \delta^{t-1} \gamma^{-(t-1)\rho} \gamma^{t-1} = \sum_{t=1}^{\infty} \left[\delta \gamma^{1-\rho}\right]^t = \frac{\delta \gamma^{1-\rho}}{1 - \delta \gamma^{1-\rho}}$$

to be finite we need $\delta \gamma^{-\rho} < 1$

$$\frac{\partial}{\partial \gamma} \frac{\delta \gamma^{1-\rho}}{1 - \delta \gamma^{1-\rho}} = \frac{\delta (1-\rho) \gamma^{-\rho}}{(1 - [\delta \gamma^{-\rho}])^2}$$

$\rho > 1$ this is negative

Hyperbolic Discounting

(based on Villaverde and Mukherji [2001])

Q1: would you like \$10 today or \$15 tomorrow?

Q2: would you like \$10 100 days from now or \$14 101 days from now?

Some people answer prefer \$10 in Q1 and \$14 in Q2. This is inconsistent with (geometric) discounting and a time and risk invariant marginal rate of substitution between days.

Note that (because of asset markets) this makes little sense when expressed in terms of money. So let us suppose that the "paradox" refers to consumption.

One explanation: "hyperbolic discounting" meaning preferences of the form $u(c_1) + \theta \sum_{t=2}^{\infty} \delta^{t-1} u(c_t)$

Another Explanation

Uncertainty about preferences 100 days from now.

Suppose marginal utility of consumption can take on two values 1 or 2 with equal probability and that the daily subjective discount factor is to a good approximation 1.

Today the value of todays and tomorrow's marginal utility is know with certainty. Hence the subjective interest rate can take on the values of 1, 0 or -½ with probabilities .25, .5 and .25. Expected subjective interest rate is .125 = 1/8. If you are offered 10 today versus 15 tomorrow, you take 10 today with probability .25.

Suppose on the other hand, suppose that preferences 100 days from now are unknown. Ratio of expected utilities is 1, so subjective interest rate is 0. If you are offered 10 in 100 days versus 14 in 101 days you always take 14.

Notice that pigeons have apparently figured this out correctly.

Demand for commitment? 13%

Time and Uncertainty

able 1 – Dynamic Preference Reversal

		Probability of reward[1]	
Scenario		1.0 (60)	0.5 (100)
1	S $175 now	0.82	0.39
	L $192 4 weeks	0.18	0.61
2	S $175 26 weeks	0.37	0.33
	L $192 30 weeks	0.63	0.67

Keren, G. and P. Roelofsma [1995], "Immediacy and Certainty in Intertemporal Choice," *Organizational Behavior and Human Decision Making*, **63** 297-297.

[1] Sample size in parentheses.

Interpersonal Preferences

Experimental results

Roth et al [1991]

US $10.00 stake games, round 10

Second and final round of bargaining game:

Player may take x or reject it and get nothing.

The other player gets $10-x

5 of 27 offers with x>0 are rejected

5 of 14 offers with 5>x>0 are rejected

x	Offers	Rejection Probability
$2.00	1	100%
$3.25	2	50%
$4.00	7	14%
$4.25	1	0%
$4.50	2	100%
$4.75	1	0%
$5.00	13	0%
total	27	

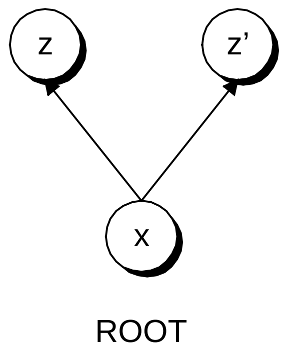
Dynamic Games

Definition of Extensive Form Game

a finite game tree X with nodes $x \in X$

nodes are partially ordered and have a single root (minimal element)

terminal nodes are $z \in Z$ (maximal elements)



Players and Information Sets

player 0 is nature

information sets $h \in H$ are a partition of $X \setminus Z$

each node in an information set must have exactly the same number of immediate followers

each information set is associated with a unique player who "has the move" at that information set

$H_i \subset H$ information sets where *i* has the move

More Extensive Form Notation

information sets belonging to nature $h \in H_0$ are singletons

$A(h)$ feasible actions at $h \in H$

each action and node $a \in A(h), x \in h$ is associated with a unique node that immediately follows x on the tree

each terminal node has a payoff $r_i(z)$ for each player

by convention we designate terminal nodes in the diagram by their payoffs

Behavior Strategies

a *pure strategy* is a map from information sets to feasible actions $s_i(h_i) \in A(h_i)$

a *behavior strategy* is a map from information sets to probability distributions over feasible actions $\pi_i(h_i) \in P(A(h_i))$

Nature's move is a behavior strategy for Nature and is a fixed part of the description of the game

We may now define $u_i(\pi)$

normal form are the payoffs $u_i(s)$ derived from the game tree

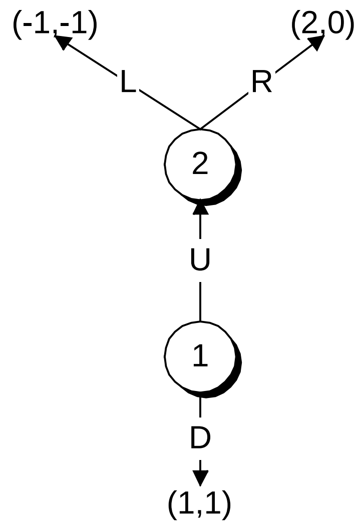
Kuhn's Theorem: every mixed strategy gives rise to a unique behavior strategy; The converse is NOT true

Subgame Perfection

A *subgame perfect Nash Equilibrium is a Nash equilibrium in every subgame*

A subgame starts at a singleton information set

Selten Game



	L	R
U	-1,-1	2,0 (SGP)
D	1,1(Nash)	1,1

- trembling hand perfection

Agent Normal Form

each information set is treated as a different player, for example 1a, 1b if player 1 has two information sets; players 1a and 1b have the same payoffs as player 1

extensive form trembling hand perfection is trembling hand perfection in the agent normal form

what is sequentiality??

Sequentiality

Kreps-Wilson [1982]

Subforms

Beliefs: *assessment* a_i for player *i* probability distribution over nodes at each of his information sets; *belief* for player *i* is a pair $b_i \equiv (a_i, \pi^i_{-i})$, consisting of *i*'s assessment over nodes a_i, and *i*'s expectations of opponents' strategies π^i_{-i}.

Beliefs come from strictly positive perturbations of strategies

belief $b_i \equiv (a_i, \pi^i_{-i})$ is *consistent* (Kreps and Wilson [17]) if $a_i = \lim_{n\to\infty} a_i^n$ where a_i^n obtained using Bayes rule on a sequence of strictly positive strategy profiles of the opponents, $\pi^{i,m}_{-i} \to \pi_{-i}$

given beliefs we have a well-defined decision problem at each information set; can define optimality at each information set

A sequential equilibrium is a behavior strategy profile π and an assessment a_i for each player such that (a_i, π^i_{-i}) is consistent and each player optimizes at each information set

Types

Harsanyi [1967]

- What happens when players do not know one another's payoffs?
- Games of "incomplete information" versus games of "imperfect information"
- Harsanyi's notion of "types" encapsulating "private information"
- Nature moves first and assigns each player a type; player's know their own types but not their opponents' types
- Players do have a common prior belief about opponents' types

Bayesian Games

There are a finite number of types $\theta_i \in \Theta_i$

There is a common prior $p(\theta)$ shared by all players

$p(\theta_{-i} \mid \theta_i)$ is the conditional probability a player places on opponents' types given his own type

The *stage* game has finite action spaces $a_i \in A_i$ and has utility functions $u^i(a,\theta)$

Bayesian Equilibrium

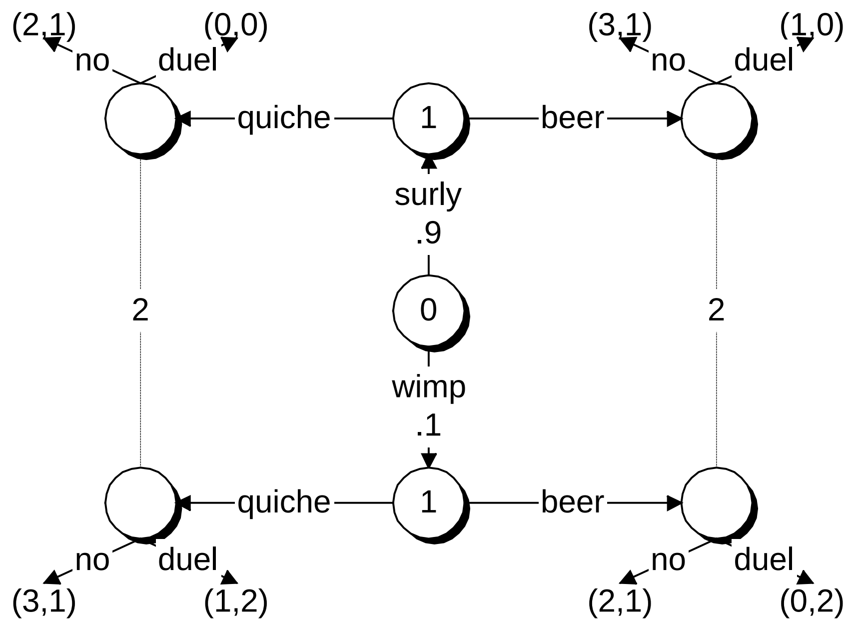
A *Bayesian Equilibrium* is a Nash equilibrium of the game in which the strategies are maps from types $s_i : \Theta_i \to A_i$ to stage game actions A_i

This is equivalent to each player having a strategy as a function of his type $s_i(\theta_i)$ that maximizes conditional on his own type θ_i (for each type that has positive probability)

$$\max_{s_i} \sum_{\theta_{-i}} u_i(s_i, s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) p(\theta_{-i} \mid \theta_i)$$

Sequentiality and Signaling

Cho-Kreps [1987]



Self Confirming Equilibrium

$\bar{H}(\sigma)$ reached with positive probability under σ

$\hat{\pi}(h_i|\sigma_i)$ map from mixed to behavior strategies

μ_i a probability measure on Π_{-i}

$u_i(s_i|\mu_i)$ preferences

$$\Pi_{-i}(\sigma_{-i}|J) \equiv \{\pi_{-i}|\pi_i(h_i) = \hat{\pi}(h_i|\sigma_i), \forall h_i \in H_{-i} \cap J\}$$

Notions of Equilibrium

Nash equilibrium

a mixed profile σ such that for each $s_i \in \mathrm{supp}(\sigma_i)$ there exist beliefs μ_i such that

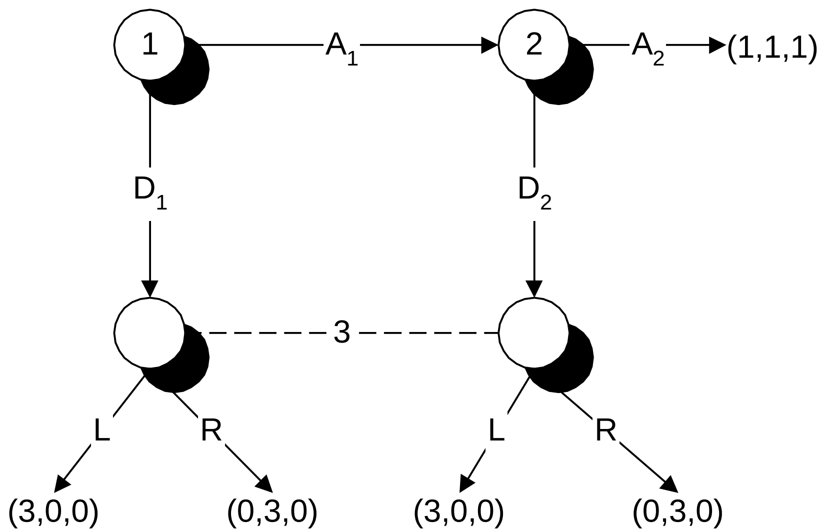
- s_i maximizes $u_i(\cdot|\mu_i)$
- $\mu_i(\Pi_{-i}(\sigma_{-i}|H)) = 1$

Unitary Self-Confirming Equilibrium

- $\mu_i(\Pi_{-i}(\sigma_{-i} \mid \bar{H}(\sigma))) = 1$

(=Nash with two players)

Fudenberg-Kreps Example



A_1, A_2 is self-confirming, but not Nash

any strategy for 3 makes it optimal for either 1 or 2 to play down

but in self-confirming, 1 can believe 3 plays R; 2 that he plays L

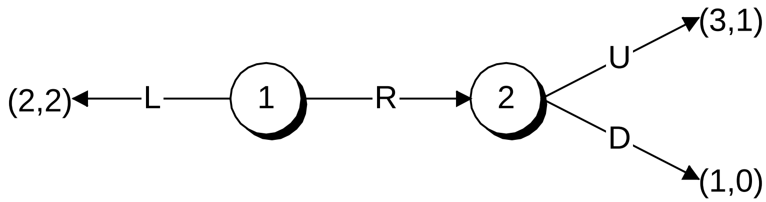
Heterogeneous Self-Confirming equilibrium

- $\mu_i(\Pi_{-i}(\sigma_{-i}|\overline{H}(s_i,\sigma))) = 1$

Can summarize by means of "observation function"

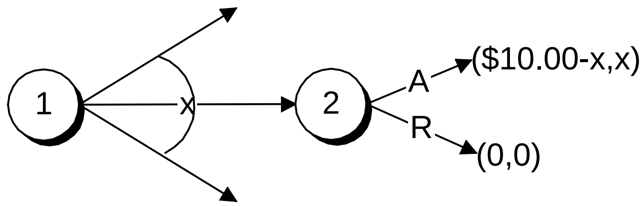
$$J(s_i,\sigma) = H, \overline{H}(\sigma), \overline{H}(s_i,\sigma)$$

Public Randomization



Remark: In games with perfect information, the set of heterogeneous self-confirming equilibrium payoffs (and the probability distributions over outcomes) are convex

Ultimatum Bargaining Results



Raw US Data for Ultimatum

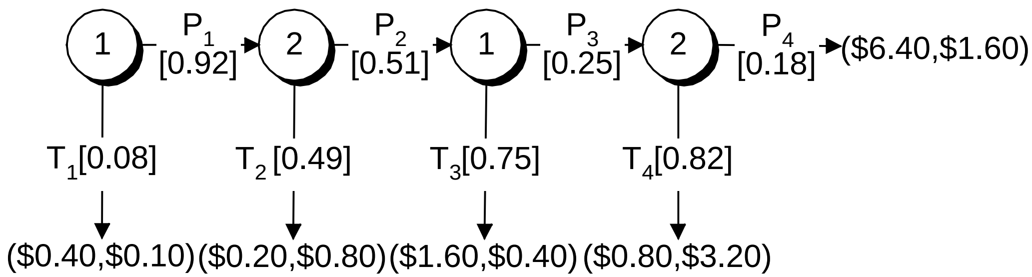
x	Offers	Rejection Probability
$2.00	1	100%
$3.25	2	50%
$4.00	7	14%
$4.25	1	0%
$4.50	2	100%
$4.75	1	0%
$5.00	13	0%
	27	

US $10.00 stake games, round 10

Trials	Rnd	Cntry Stake	Case	Expected Loss			Max Gain	Ratio
				Pl 1	Pl 2	Both		
27	10	US	H	$0.00	$0.67	$0.34	$10.00	3.4%
27	10	US	U	$1.30	$0.67	$0.99	$10.00	9.9%
10	10	USx3	H	$0.00	$1.28	$0.64	$30.00	2.1%
10	10	USx3	U	$6.45	$1.28	$3.86	$30.00	12.9%
30	10	Yugo	H	$0.00	$0.99	$0.50	$10?	5.0%
30	10	Yugo	U	$1.57	$0.99	$1.28	$10?	12.8%
29	10	Jpn	H	$0.00	$0.53	$0.27	$10?	2.7%
29	10	Jpn	U	$1.85	$0.53	$1.19	$10?	11.9%
30	10	Isrl	H	$0.00	$0.38	$0.19	$10?	1.9%
30	10	Isrl	U	$3.16	$0.38	$1.77	$10?	17.7%
	WC		H			$5.00	$10.00	50.0%

Rnds=Rounds, WC=Worst Case, H=Heterogeneous, U=Unitary

Centipede Game: Palfrey and McKelvey



Numbers in square brackets correspond to the observed conditional probabilities of play corresponding to rounds 6-10, stakes 1x below.

This game has a unique self-confirming equilibrium; in it player 1 with probability 1 plays T_1

Summary of Experimental Results

Trials / Rnd	Rnds	Stake	Case	Expected Loss			Max Gain	Ratio
				Pl 1	Pl 2	Both		
29*	6-10	1x	H	$0.00	$0.03	$0.02	$4.00	0.4%
29*	6-10	1x	U	$0.26	$0.17	$0.22	$4.00	5.4%
	WC	1x	H			$0.80	$4.00	20.0%
29	1-10	1x	H	$0.00	$0.08	$0.04	$4.00	1.0%
10	1-10	4x	H	$0.00	$0.28	$0.14	$16.00	0.9%

Rnds=Rounds, WC=Worst Case, H=Heterogeneous, U=Unitary

*The data on which from which this case is computed is reported above.

Learning and Self-confirming Equilibrium

government chooses high or low inflation…then in the next stage

consumers choose high or low unemployment; but prefers low unemployment

government gets 2 for low unemployment plus 1 for low inflation

subgame-perfect equilibrium: government chooses low inflation and gets 3

self-confirming equilibrium: government believes that low inflation leads to high unemployment, so chooses high inflation and gets 2

no data is generated about the consequences of low inflation

Sargent, Williams, Zhao 2006: detailed explanation of how learning by the U.S. Federal Reserve led to the conquest of American inflation

The Ordinary, the Extraordinary and the Dishonest

Periodic short crises during which long-run beliefs of consumers are wrong, although short-run beliefs are right

Sargent, Williams, Zha 2008

- The current crisis: the ordinary; the extraordinary and the dishonest