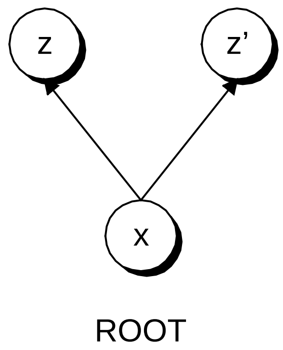
Dynamic Games

Definition of Extensive Form Game

a finite game tree X with nodes $x \in X$

nodes are partially ordered and have a single root (minimal element)

terminal nodes are $z \in Z$ (maximal elements)



Players and Information Sets

player 0 is nature

information sets $h \in H$ are a partition of $X \setminus Z$

each node in an information set must have exactly the same number of immediate followers

each information set is associated with a unique player who “has the move” at that information set

$H_i \subset H$ information sets where *i* has the move

More Extensive Form Notation

information sets belonging to nature $h \in H_0$ are singletons

$A(h)$ feasible actions at $h \in H$

each action and node $a \in A(h), x \in h$ is associated with a unique node that immediately follows x on the tree

each terminal node has a payoff $r_i(z)$ for each player

by convention we designate terminal nodes in the diagram by their payoffs

Behavior Strategies

a *pure strategy* is a map from information sets to feasible actions $s_i(h_i) \in A(h_i)$

a *behavior strategy* is a map from information sets to probability distributions over feasible actions $\pi_i(h_i) \in P(A(h_i))$

Nature's move is a behavior strategy for Nature and is a fixed part of the description of the game

We may now define $u_i(\pi)$

normal form are the payoffs $u_i(s)$ derived from the game tree

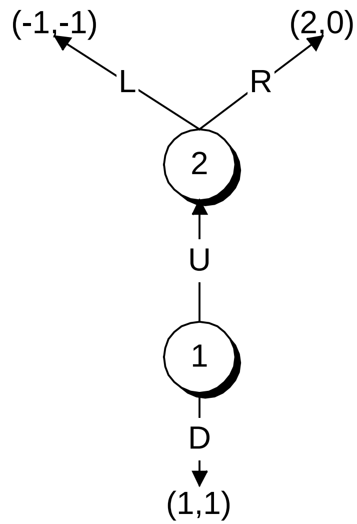
Kuhn's Theorem: every mixed strategy gives rise to a unique behavior strategy; The converse is NOT true

Subgame Perfection

A *subgame perfect Nash Equilibrium is a Nash equilibrium in every subgame*

A subgame starts at a singleton information set

Selten Game



	L	R
U	-1,-1	2,0 (SGP)
D	1,1(Nash)	1,1

- trembling hand perfection

Agent Normal Form

each information set is treated as a different player, for example 1a, 1b if player 1 has two information sets; players 1a and 1b have the same payoffs as player 1

extensive form trembling hand perfection is trembling hand perfection in the agent normal form

what is sequentiality??

Sequentiality

Kreps-Wilson [1982]

Subforms

Beliefs: *assessment* a_i for player *i* probability distribution over nodes at each of his information sets; *belief* for player *i* is a pair $b_i \equiv (a_i, \pi^i_{-i})$, consisting of *i*'s assessment over nodes a_i, and *i*'s expectations of opponents' strategies π^i_{-i}.

Beliefs come from strictly positive perturbations of strategies

belief $b_i \equiv (a_i, \pi^i_{-i})$ is *consistent* (Kreps and Wilson [17]) if $a_i = \lim_{n\to\infty} a_i^n$ where a_i^n obtained using Bayes rule on a sequence of strictly positive strategy profiles of the opponents, $\pi^{i,m}_{-i} \to \pi_{-i}$

given beliefs we have a well-defined decision problem at each information set; can define optimality at each information set

A sequential equilibrium is a behavior strategy profile π and an assessment a_i for each player such that (a_i, π^i_{-i}) is consistent and each player optimizes at each information set

Types

Harsanyi [1967]

- What happens when players do not know one another's payoffs?
- Games of "incomplete information" versus games of "imperfect information"
- Harsanyi's notion of "types" encapsulating "private information"
- Nature moves first and assigns each player a type; player's know their own types but not their opponents' types
- Players do have a common prior belief about opponents' types

Bayesian Games

There are a finite number of types $\theta_i \in \Theta_i$

There is a common prior $p(\theta)$ shared by all players

$p(\theta_{-i} \mid \theta_i)$ is the conditional probability a player places on opponents' types given his own type

The *stage* game has finite action spaces $a_i \in A_i$ and has utility functions $u^i(a,\theta)$

Bayesian Equilibrium

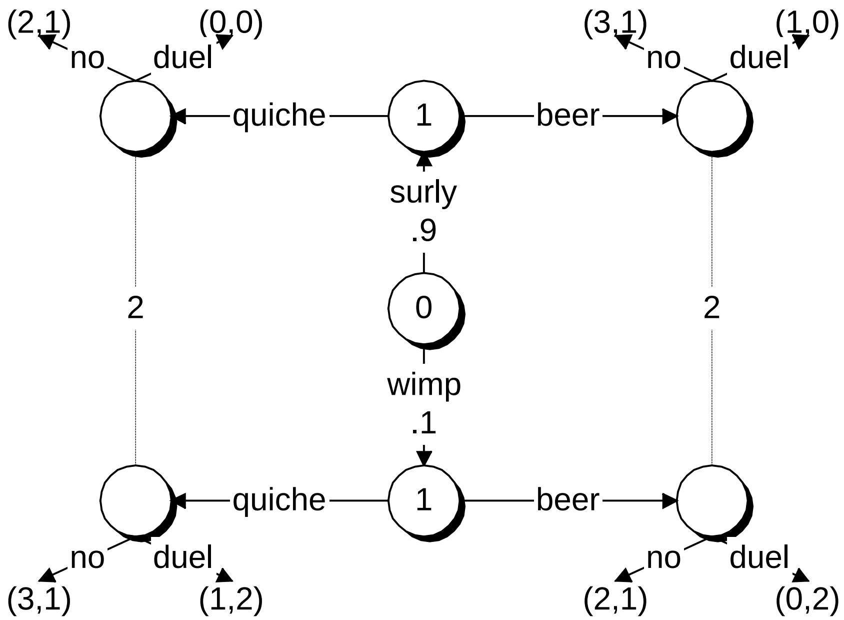
A *Bayesian Equilibrium* is a Nash equilibrium of the game in which the strategies are maps from types $s_i : \Theta_i \rightarrow A_i$ to stage game actions A_i

This is equivalent to each player having a strategy as a function of his type $s_i(\theta_i)$ that maximizes conditional on his own type θ_i (for each type that has positive probability)

$$\max_{s_i} \sum_{\theta_{-i}} u_i(s_i, s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) p(\theta_{-i} \mid \theta_i)$$

Sequentiality and Signaling

Cho-Kreps [1987]



Self Confirming Equilibrium

$\bar{H}(\sigma)$ reached with positive probability under σ

$\hat{\pi}(h_i | \sigma_i)$ map from mixed to behavior strategies

μ_i a probability measure on Π_{-i}

$u_i(s_i | \mu_i)$ preferences

$$\Pi_{-i}(\sigma_{-i} | J) \equiv \{\pi_{-i} | \pi_i(h_i) = \hat{\pi}(h_i | \sigma_i), \forall h_i \in H_{-i} \cap J\}$$

Notions of Equilibrium

Nash equilibrium

a mixed profile σ such that for each $s_i \in \operatorname{supp}(\sigma_i)$ there exist beliefs μ_i such that

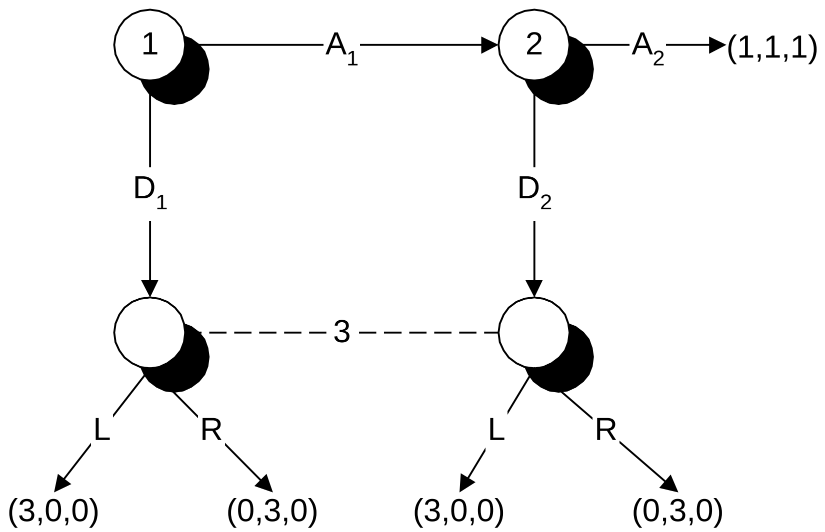
- s_i maximizes $u_i(\cdot|\mu_i)$
- $\mu_i(\Pi_{-i}(\sigma_{-i}|H)) = 1$

Unitary Self-Confirming Equilibrium

- $\mu_i(\Pi_{-i}(\sigma_{-i} \mid \overline{H}(\sigma))) = 1$

(=Nash with two players)

Fudenberg-Kreps Example



A_1, A_2 is self-confirming, but not Nash

any strategy for 3 makes it optimal for either 1 or 2 to play down

but in self-confirming, 1 can believe 3 plays R; 2 that he plays L

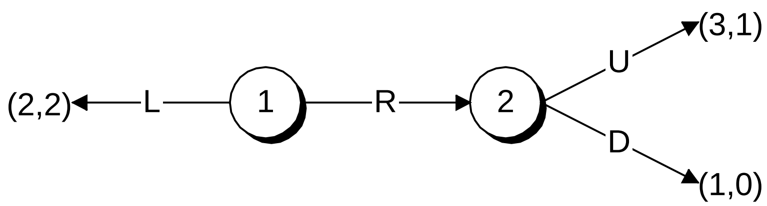
Heterogeneous Self-Confirming equilibrium

- $\mu_i(\Pi_{-i}(\sigma_{-i} | \overline{H}(s_i, \sigma))) = 1$

Can summarize by means of “observation function”

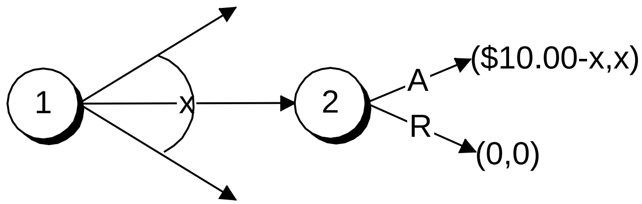
$$J(s_i, \sigma) = H, \overline{H}(\sigma), \overline{H}(s_i, \sigma)$$

Public Randomization



Remark: In games with perfect information, the set of heterogeneous self-confirming equilibrium payoffs (and the probability distributions over outcomes) are convex

Ultimatum Bargaining Results



Raw US Data for Ultimatum

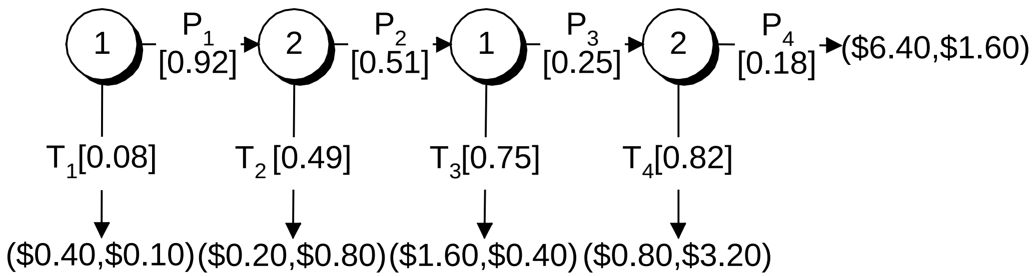
x	Offers	Rejection Probability
$2.00	1	100%
$3.25	2	50%
$4.00	7	14%
$4.25	1	0%
$4.50	2	100%
$4.75	1	0%
$5.00	13	0%
	27	

US $10.00 stake games, round 10

Trials	Rnd	Cntry Stake	Case	Expected Loss			Max Gain	Ratio
				Pl 1	Pl 2	Both		
27	10	US	H	$0.00	$0.67	$0.34	$10.00	3.4%
27	10	US	U	$1.30	$0.67	$0.99	$10.00	9.9%
10	10	USx3	H	$0.00	$1.28	$0.64	$30.00	2.1%
10	10	USx3	U	$6.45	$1.28	$3.86	$30.00	12.9%
30	10	Yugo	H	$0.00	$0.99	$0.50	$10?	5.0%
30	10	Yugo	U	$1.57	$0.99	$1.28	$10?	12.8%
29	10	Jpn	H	$0.00	$0.53	$0.27	$10?	2.7%
29	10	Jpn	U	$1.85	$0.53	$1.19	$10?	11.9%
30	10	Isrl	H	$0.00	$0.38	$0.19	$10?	1.9%
30	10	Isrl	U	$3.16	$0.38	$1.77	$10?	17.7%
	WC		H			$5.00	$10.00	50.0%

Rnds=Rounds, WC=Worst Case, H=Heterogeneous, U=Unitary

Centipede Game: Palfrey and McKelvey



Numbers in square brackets correspond to the observed conditional probabilities of play corresponding to rounds 6-10, stakes 1x below.

This game has a unique self-confirming equilibrium; in it player 1 with probability 1 plays T_1

Summary of Experimental Results

Trials / Rnd	Rnds	Stake	Case	Expected Loss			Max Gain	Ratio
				Pl 1	Pl 2	Both		
29*	6-10	1x	H	$0.00	$0.03	$0.02	$4.00	0.4%
29*	6-10	1x	U	$0.26	$0.17	$0.22	$4.00	5.4%
	WC	1x	H			$0.80	$4.00	20.0%
29	1-10	1x	H	$0.00	$0.08	$0.04	$4.00	1.0%
10	1-10	4x	H	$0.00	$0.28	$0.14	$16.00	0.9%

Rnds=Rounds, WC=Worst Case, H=Heterogeneous, U=Unitary

*The data on which from which this case is computed is reported above.

Learning and Self-confirming Equilibrium

government chooses high or low inflation…then in the next stage

consumers choose high or low unemployment; but prefers low unemployment

government gets 2 for low unemployment plus 1 for low inflation

subgame-perfect equilibrium: government chooses low inflation and gets 3

self-confirming equilibrium: government believes that low inflation leads to high unemployment, so chooses high inflation and gets 2

no data is generated about the consequences of low inflation

Sargent, Williams, Zhao 2006: detailed explanation of how learning by the U.S. Federal Reserve led to the conquest of American inflation

The Ordinary, the Extraordinary and the Dishonest

Periodic short crises during which long-run beliefs of consumers are wrong, although short-run beliefs are right

Sargent, Williams, Zha 2008

- The current crisis: the ordinary; the extraordinary and the dishonest