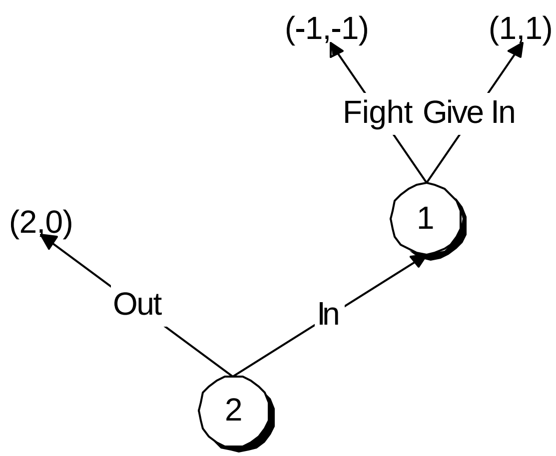
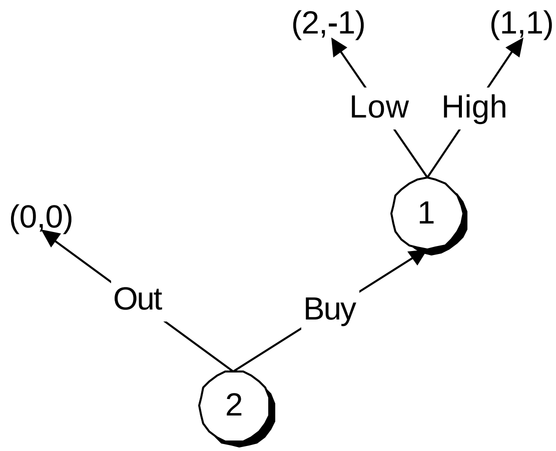
Reputation

Extensive Form Examples



Chain Store Game



Quality Game

Simultaneous Move Examples

Modified Chain Store

	out	in
fight	2-e, 0	-1,-1
give in	2,0	1,1**

Inflation Game

	Low	High
Low	0,0	-2,-1
High	1,-1	-1,0

Inflation Game: LR=government, SR=consumers

consumer preferences are whether or not they guess right

	Low	High
Low	0,0	0,-1
High	-1,-1	-1,0

with a hard-nosed government

The Model

multiple types of long-run player $w \in \Omega$

Ω is a countable set of types

type is fixed forever (does not change from period to period)

$u^1(a,w)$ utility depends on type

strategy $s^1(h,w)$ depends on type

types are privately known to long-run player, not known to short run player

strategy $s^2(h)$ does not depend on type

m probability distribution over Ω commonly known short-run player prior over types

Truly Committed Types

type $w(a^1)$ has a dominant strategy to play a^1 in the repeated game:

$$u^1(\tilde{a}^1, a^2, w(a^1)) = \begin{cases} 1 & \tilde{a}^1 = a^1 \\ 0 & \tilde{a}^1 \neq a^1 \end{cases}$$

for example

Let $n(w)$ be the least utility received by a type w in any Nash equilibrium

let $a^1 *$ be a pure strategy Stackelberg strategy for type w_0, with corresponding utility

$$u^1 * = \max_{a^1} \min_{a^2 \in BR(a^1)} u^1(a^1, a^2, w_0)$$

Theorem: Fix w_0 with $m(w_0) > 0$, and. Let $w^* \equiv w(a^1*)$, and suppose that $m^* \equiv m(w^*) > 0$. Then there is a constant $k(m^*)$ otherwise independent of m, Ω such that

$$n(w) \geq d^{k(m^*)} u^1 * + (1 - d^{k(m^*)}) \underline{u}^1$$

Proof

define $p_t{*}$ to be the probability at the beginning of period *t* by the short-run player that he is facing type $w{*}$

Let $N(p_t{*} \le \overline{p})$ be the number of times $p_t{*} \le \overline{p}$

Lemma 1: Suppose that LR plays $a^1{*}$ always. Then for any history *h* that has positive probability

$$pr(N(p_t{*} \le \overline{p}) > \log m{*}/\log \overline{p} | h) = 0$$

Lemma 2: There is $\overline{p} < 1$ such that if $p_t{*} > \overline{p}$ the SR player plays a best response to $a^1{*}$

- Why do these Lemma’s imply the theorem?

- Why is Lemma 2 true?

Proof of Lemma 1

Bayes Law

$$p(w^*|h_t) = \frac{p(w^*|h_{t-1})p(h_t|w^*,h_{t-1})}{p(h_t|h_{t-1})}$$

given h_{t-1} player 1 and 2 play independently

$$p(w^*|h_t) = \frac{p(w^*|h_{t-1})p(h_t|w^*,h_{t-1})}{p(h_t^1|h_{t-1})p(h_t^2|h_{t-1})}$$

since player 1's type isn't known to player 2

$$p(w^*|h_t) = \frac{p(w^*|h_{t-1})p(h_t|w^*,h_{t-1})}{p(h_t^1|h_{t-1})p(h_t^2|w^*,h_{t-1})}$$

since player 1's strategy is to always play a^1* $p(h_t^1|w^*,h_{t-1}) = 1$ so

$$p(w^*|h_t) = \frac{p(w^*|h_{t-1})p(h_t^2|w^*,h_{t-1})}{p(h_t^1|h_{t-1})p(h_t^2|w^*,h_{t-1})}$$

$$= \frac{p(w^*|h_{t-1})}{p(h_t^1|h_{t-1})} = \frac{p(w^*|h_{t-1})}{p_t^*}$$

the conclusion:

$$p(w^*|h_t) = \frac{p(w^*|h_{t-1})}{p_t^*}$$

- what does this say?

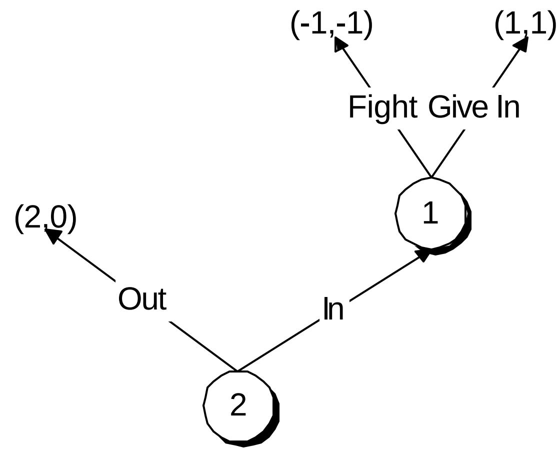
the Lemma now derives from the fact that $p(w^*|h_t) \leq 1$

Observational Equivalence

$r(y|a)$ outcome function

$a^2 \in W(a^1)$ if there exists $\tilde{a}^1$ such that $r(\cdot|\tilde{a}^1, a^2) = r(\cdot|a^1, a^2)$ and $a^2 \in BR(\tilde{a}^1)$

$$u^{1*} = \max_{a^1} \min_{a^2 \in W(a^1)} u^1(a^1, a^2, w_0)$$



Chain Store Game

strategies that are observationally equivalent

	out	in	mixed
fight	all	fight	fight
give	all	give	give
mixed	all	mixed	mixed

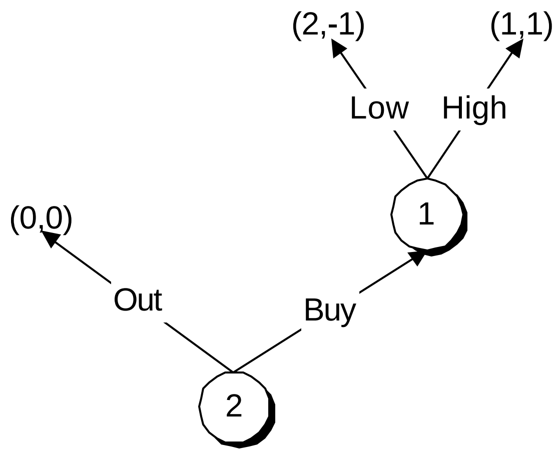
weak best responses

fight: out

give: in, out

mixed: in, out?

Best case fight:out so $u^1* = 2$



Quality Game

strategies that are observationally equivalent

	out	buy	mixed
hi	all	hi	hi
lo	all	lo	lo
mixed	all	mixed	mixed

weak best responses

hi: in, out

lo: out

mixed: in?, out

in every case out is a weak best response so $u^1* = 0$

Moral Hazard and Mixed Commitments

$r(y|a)$ outcome function

expand space of types to include types committed to mixed strategies: leads to technical complications because it requires a continuum of types

$p(h_{t-1})$ probability distribution over outcomes conditional on the history (a vector)

$p^+(h_{t-1})$ probability distribution over outcomes conditional on the history and the type being in Ω^+

Theorem: for every $e > 0, \Delta_0 > 0$ and set of types Ω^+ with $m(\Omega^+) > 0$ there is a *K* such that if Ω^+ is true there is probability less than e that there are more than *K* periods with

$$\left\| p^+(h_{t-1}) - p(h_{t-1}) \right\| > \Delta_0$$

look for tight bounds

let $\underline{n}, \overline{n}$ be best and worst Nash payoffs to LR

try to get

$$\liminf_{d \to 1} \underline{n}(w) = \limsup_{d \to 1} \overline{n}(w) = \max_{a^2 \in BR(a^1)} u^1(a)$$

game is *non-degenerate* if there is no undominated pure action a^2 such that for some $a^2 \neq a^2$

$$u^i(\cdot, a^2) = u^i(\cdot, a^2)$$

counterexample: player 2 gets zero always, player 1 gets either zero or one depending only on player 2's action game is *identified* if for all a^2 that are not weakly dominated $r(\cdot|a^1,a^2) = r(\cdot|\tilde{a}^1,a^2)$ implies $a^1 = \tilde{a}^1$

$$r(\cdot|a^1,a^2) = a^1 R(a^2)$$

condition for identification $R(a^2)$ has full row rank for all a^2

Patient Short Run Players: Schmidt

short run preferences $\begin{pmatrix} 10 & 0 \\ 0 & 1 \end{pmatrix}$

long run preferences

$m^0 = 0.1$ $\begin{pmatrix} 10 & 0 \\ 0 & 1 \end{pmatrix}$ pure coordination

$m^* = 0.01$ $\begin{pmatrix} 10 & 10 \\ 0 & 1 \end{pmatrix}$ commitment type

$m^i = 0.89$ $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ indifferent type strategies:

normal: play U except if you previously did D, then switch to D

commitment: always play U

indifferent type: U until deviation then D

SR: play L then alternate between R and L (on path)

if 1 deviated to D switch to R forever

if 2 deviated play L; if 1 reacts with U continue with L

reacts with D continue with R

$d_1 \geq .15, d_2 \geq .75$ then this is a subgame perfect equilibrium

- interesting deviation for SR when supposed to do R deviate to L; but then indifferent type switches to D forever
- for the normal type to prove he’s not type “i” he must play D revealing he is not the commitment type

Suppose that LR can minmax SR in a pure strategy $\underline{a}^1$

Theorem: LR gets at least $\min_{a^2 \in BR^2(\underline{a}^1)} u^1(\underline{a}^1, a^2)$

let $\underline{u}^2$ be SR minmax

let $\tilde{u}^2$ be second best against $\underline{a}^1$

$$N = \frac{\ln(1-d_2) + \ln(\underline{u}^2 - \tilde{u}^2) - n(\overline{u}^2 - \tilde{u}^2)}{\ln d_2}$$

$$e = \frac{(1-d_2)^2(\underline{u}^2 - \tilde{u}^2)}{(\overline{u}^2 - \tilde{u}^2)} - d_2{}^N(1-d_2)$$

commit to $\underline{a}^1$

Lemma: suppose $a_2^{t+1} \notin BR(\underline{a}^1)$ with positive probability, then SR must believe that in t+1,…,t+N there is a probability of at least e of not having $\underline{a}^1$

- why is this sufficient?

Proof of Lemma:

2 can get at least $\underline{u}^2$ so

$$(1-d_2)u(a^1, a_2^{t+1}) + d_2 V \geq \underline{u}^2$$

if $pr(\underline{a}^1) > 1-e$ in t+1,…,t+N

lose at least $\underline{u}^2 - \tilde{u}^2$ at t+1

in rest of game gain at most

$$(1-d_2)\sum_{t=1}^{N} d_2^t(\underline{u}^2(1-e) + e\overline{u}^2) + d_2^{N+1}\overline{u}^2$$

but we chose *N* and e so that the loss exceeds the gain