

# **The Paradox of Voter Participation? A Laboratory Study**

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## *Introduction*

- rational choice theory of turnout originally formulated by Downs (1957)
- extremely controversial formal theory in political science
- in its purest form grossly under predicts turnout rates in mass elections
- act of voting one of the least well understood phenomena in study of politics
- why do some people vote and others not?
- What affects turnout rates?
- differences between poll predictions and actual electoral outcomes often accounted for by a failure to accurately predict turnout

## *``the paradox of voter turnout''*

- kindled debate about value of rational choice approach to study of political behavior
- Green and Shapiro (1994) use it as poster boy for critics of ``homo economicus'' approach to political science
- Fiorina (1989) [sympathetic to the rational choice approach] dubbed it ``the paradox that ate rational choice theory.''
- modifications to basic model lead to correction of underprediction
- criticized as ad hoc and narrowly tailored to voter turnout problem
- empirical work examines the comparative static predictions of theory
- too little control over the distribution of voter preferences and voting costs to give much power to tests

## *Comparative Statics of Voter Rational Choice Voter Turnout*

we focus three main comparative static results

- *size effect*: increase in size of the electorate leads to lower turnout rates
- *competition effect*: turnout higher in elections that are expected to be closer if everyone voted
- *underdog effect*: turnout among supporters of more popular candidate or party less than turnout among supporters of the less popular candidate

true only if voter information, distribution of voting costs, candidate quality, weather, election technology, age, income, education, voter accessibility to poll sites, and on and on all held constant

which is why empirical studies are hard

## *Why laboratory experiments?*

- laboratory experiments fully control for the preferences, voter information, costs, electorate size, and competitiveness
- choose voting environment that, for very large elections, would predict very low turnout
- experiments large by the standards of laboratory experiments - can obtain variation in predicted turnout rates
- choose parameters that imply a unique quantitative prediction

## *Literature*

Schram and Sonnemans (1996)

Cason and Mui (2005)

Grosser, Kugler and Schram (2005)

homogeneous voting costs: implies a plethora of mixed strategy equilibria

difficult to determine how well the theory performs

## *The Model*

based on the Palfrey and Rosenthal (1985)

$N$  voters, divided into two groups

supporters of candidate A, supporters of candidate B

drop the Palfrey and Rosenthal assumption that two groups are equal size

size of group A is  $N_A$  (minority, underdog); the size of group B

(majority, frontrunner) is  $N_B$  with  $N_A < N_B$

sizes common knowledge to voters

voting rule simple plurality

voters decide simultaneously to vote for preferred candidate or abstain

candidate with votes wins election, ties broken randomly

A wins all members of group A receive reward of  $H$ ; all members of group B receive reward  $L < H$

B wins all members of group B receive reward of  $H$ ; all members of group A receive reward  $L < H$

rewards are common knowledge

voting is costly, and the voting cost to voter  $i$  is denoted  $c_i$ ; private information to voter  $i$

distribution from which costs are drawn has density  $f(c)$  positive on its support and is common knowledge

quasi-symmetric cutpoint equilibrium of voting game given by a pair of numbers  $(c_A^*, c_B^*)$

a cutpoint strategy for voter  $i$  specifies that voter  $i$  abstains if and only if

$$c_i > c_i^*$$

quasi-symmetric because all voters in the same party use the same cutpoint

cannot prove directly there is a unique equilibrium, but for our parameters grid search shows that numerically there is just one equilibrium

## ***The Experimental Design and Hypotheses***

primary treatment variable choice of parameters for the game

payoffs ( $L$  and  $H$  - just a normalization),  $N_A, N_B$ , and  $f$

central hypotheses are size effect, underdog effect, competition effect,  
fix  $f$  throughout the experiment vary  $N_A$  and  $N_B$

payoffs  $L=5$  and  $H=105$

$N \in \{3, 9, 27, 51\}$  (odd numbers divisible by 3)

each electorate size: two subtreatments

*landslide*  $N_B = 2N_A$  ; *tossup*  $N_B = N_A + 1$

**Table 1:** Experimental Design

$N$	$N_A$	$N_B$	#subjects	#sessions	#elections	# obs
3	1	2	51	4	850	2550
9	3	6	81	9	450	4050
9	4	5	81	9	450	4050
27	9	18	108	4	200	5400
27	13	14	108	4	200	5400
51	17	34	102	2	100	5100
51	25	24	102	2	100	5100

### *choice of cost distribution*

two considerations: one theoretical, one logistical

- wanted a unique quasi-symmetric cut point equilibrium for all of our  $N_A, N_B$  treatments
- wanted it to be easy to explain to subjects

uniform distribution of voting costs ranging from 0 to 55 satisfies both  
payoff for making or breaking a tie is 50

voters who draw costs greater than 50 have a strict dominant strategy to abstain and voters with costs equal to 50 have a weakly dominant strategy to abstain

## *logistical challenges*

1. easy to explain
2. control information flows: individuals know own costs but only distribution of costs for others
3. parameters as close to common knowledge as possible
4. wanted subjects to have an opportunity to gain experience so as to observe equilibrium not learning
5. wanted a context-free design to avoid framing effects
6. wanted a design which allowed within subject tests as well as between subject tests

## Comparative Statics Hypotheses

$p^*$  are turnout probabilities

**H1. The Size Effect.** Holding the relative size of the two groups constant, turnout in each party is decreasing in  $N$ . Formally, this implies lots of specific hypotheses in the form of pairwise inequalities. Two examples are  $p_A^*(4,5) > p_A^*(13,14)$  and  $p_B^*(1,2) > p_B^*(17,34)$ .

Generally these inequalities take the form that if  $N < M$ , then

$$p_j^*\left(\frac{N-1}{2}, \frac{N+1}{2}\right) > p_j^*\left(\frac{M-1}{2}, \frac{M+1}{2}\right) \text{ and } p_j^*\left(\frac{N}{3}, \frac{2N}{3}\right) > p_j^*\left(\frac{M}{3}, \frac{2M}{3}\right) \text{ for}$$

$j=A, B$ . In addition, there is the weaker hypothesis that total turnout is decreasing in  $N$ .

**H2. The Competition Effect.** Holding the total size of the electorate constant, turnout in each party is decreasing in  $N_A - N_B$ . This gives us 6 specific hypotheses in the form of pairwise inequalities. They are  $p_A^*(4,5) > p_A^*(3,6)$ ,  $p_A^*(13,14) > p_A^*(9,18)$ ,  $p_A^*(25,26) > p_A^*(17,34)$ ,  $p_B^*(4,5) > p_B^*(3,6)$ ,  $p_B^*(13,14) > p_B^*(9,18)$ ,  $p_B^*(25,26) > p_B^*(17,34)$ . Note that this hypothesis does not apply for the  $N=3$  treatment, since  $\frac{N-1}{2} = \frac{N}{3}$  in that case.

**H3. The Underdog Effect.** For  $N > 3$ , turnout of party  $A$  is greater than turnout of party  $B$ . This gives us 6 specific hypotheses in the form of pairwise inequalities. They are  $p_A^*(4,5) > p_B^*(4,5)$ ,  $p_A^*(3,6) > p_B^*(3,6)$ ,  $p_A^*(13,14) > p_B^*(13,14)$ ,  $p_A^*(9,18) > p_B^*(9,18)$ ,  $p_A^*(25,26) > p_B^*(25,26)$ ,  $p_A^*(17,34) > p_B^*(17,34)$ .

**H4. Counter-example to the Underdog Effect.** For  $N=3$ , turnout of party  $A$  is less than turnout of party  $B$ . Specifically,  $p_A^*(1,2) < p_B^*(1,2)$ .

*quantitative hypotheses about turnout*

$p^*$  probability of turnout

$\pi^*$  probability of a pivotal outcome

$Q^*$  probability of an upset (including a tie)

**Table 2:** Nash Equilibrium Predictions

$N$	$N_A$	$N_B$	$P_A^*$	$P_B^*$	$\pi^*$	$Q^*$
3	1	2	.537	.640	.810	.070
9	3	6	.413	.375	.599	.151
9	4	5	.460	.452	.666	.270
27	9	18	.270	.228	.409	.187
27	13	14	.302	.297	.466	.418
51	17	34	.206	.171	.309	.153
51	25	26	.238	.235	.375	.435

**H5 The size effect on the frequency of Pivotal Events.** Holding the relative size of the two groups constant, the frequency of pivotal events is decreasing in  $N$ . Formally, this implies lots of specific hypotheses in the form of pairwise inequalities. An example is  $\pi^*(4,5) > \pi^*(13,14)$ .

Generally these inequalities take the form that if  $N < M$ , then and

$$\pi^* \left( \frac{N-1}{2}, \frac{N+1}{2} \right) > \pi^* \left( \frac{M-1}{2}, \frac{M+1}{2} \right)$$

$$\pi^* \left( \frac{N}{3}, \frac{2N}{3} \right) > \pi^* \left( \frac{M}{3}, \frac{2M}{3} \right).$$

## **H6 The competition effect on the frequency of Pivotal Events.**

Holding the total size of the electorate constant, the frequency of pivotal events is decreasing in  $N_A - N_B$ . This gives us 3 specific hypotheses in

the form of pairwise inequalities:  $\pi^*(4,5) > \pi^*(3,6)$ ;

$\pi^*(13,14) > \pi^*(9,18)$ ;  $\pi^*(25,26) > \pi^*(17,34)$ .

**H7 Upset Rates.** The underdog is more likely to tie or win in toss-up races than in landslide races, and these upset probabilities are decreasing in  $N$ . This generates lots of specific hypotheses in the form of pairwise inequalities. All but three of these hypotheses are of the form  $Q^* \left( \frac{N-1}{2}, \frac{N+1}{2} \right) > Q^* \left( \frac{M-1}{2}, \frac{M+1}{2} \right)$  and  $Q^* \left( \frac{N}{3}, \frac{2N}{3} \right) > Q^* \left( \frac{M}{3}, \frac{2M}{3} \right)$  for  $N < M$ . The other three hypotheses are  $Q^* \left( \frac{N-1}{2}, \frac{N+1}{2} \right) > Q^* \left( \frac{N}{3}, \frac{2N}{3} \right)$   $N=9,27,54$ .

## *Individual behavior*

- all individuals should be using exact cutpoint rules
- obviously they don't
- consider that they follow a cutpoint rule stochastically
- follow it most of the time, but violate it some of the time
- allows us classify subjects according to their propensity to vote, which we expect to vary due to many diverse factors such as expectations about pivotal events, risk aversion, attitudes about group norms, social preferences, judgement fallacies and so forth.
- address  $H2$ ,  $H3$ , and  $H4$  at the individual level as well as at the aggregate level

## ***Experimental Protocol***

- email announcement from a subject pool consisting of registered UCLA students
- 284 different subjects
- 20 separate sessions using networked computers at the CASSEL experimental facility

- each session  $N$  was held fixed throughout the entire session
- $N > 3$  sessions, there were two subsessions of 50 rounds each
- one subsession was the toss-up treatment and the other subsession was the landslide treatment
- sequencing done both ways
- before each round, each subject assigned to either group  $A$  or group  $B$  and assigned a voting cost
- each subject gained experience as member of majority and minority party for exactly one value of  $N$ , and participated in both 50 landslide and 50 toss-up elections

instructions read aloud so everyone could hear, and Powerpoint slides were projected in front of the room to help explain the rules and to make all the common knowledge to the extent possible

after instructions were read, subjects were walked through two practice rounds with randomly forced choices and required to correctly answer all questions on a computerized comprehension

after first 50 rounds, a short set of new instructions were read aloud, explaining that the sizes of group  $A$  and group  $B$  would be different for the next 50 rounds

- wording written to induce as neutral an environment as possible
- no mention of voting or winning or losing or costs
- labels were abstract. The smaller groups was referred to the alpha group ( $A$ ) and the larger group was referred to as the beta group ( $B$ )
- asked in each round to choose X or Y
- if more members of  $A(B)$  chose X than members of  $B(A)$  chose X, then each member of  $A(B)$  received 105 and each member of group  $B(A)$  received 5
- in case of a tie, each member of each group received 55
- voting cost was referred to as a "Y bonus," and was added to a player's earnings if that player chose Y instead of X in an election.
- bonuses were randomly redrawn in every round, independently for each subject, and subjects were only told their own Y bonus

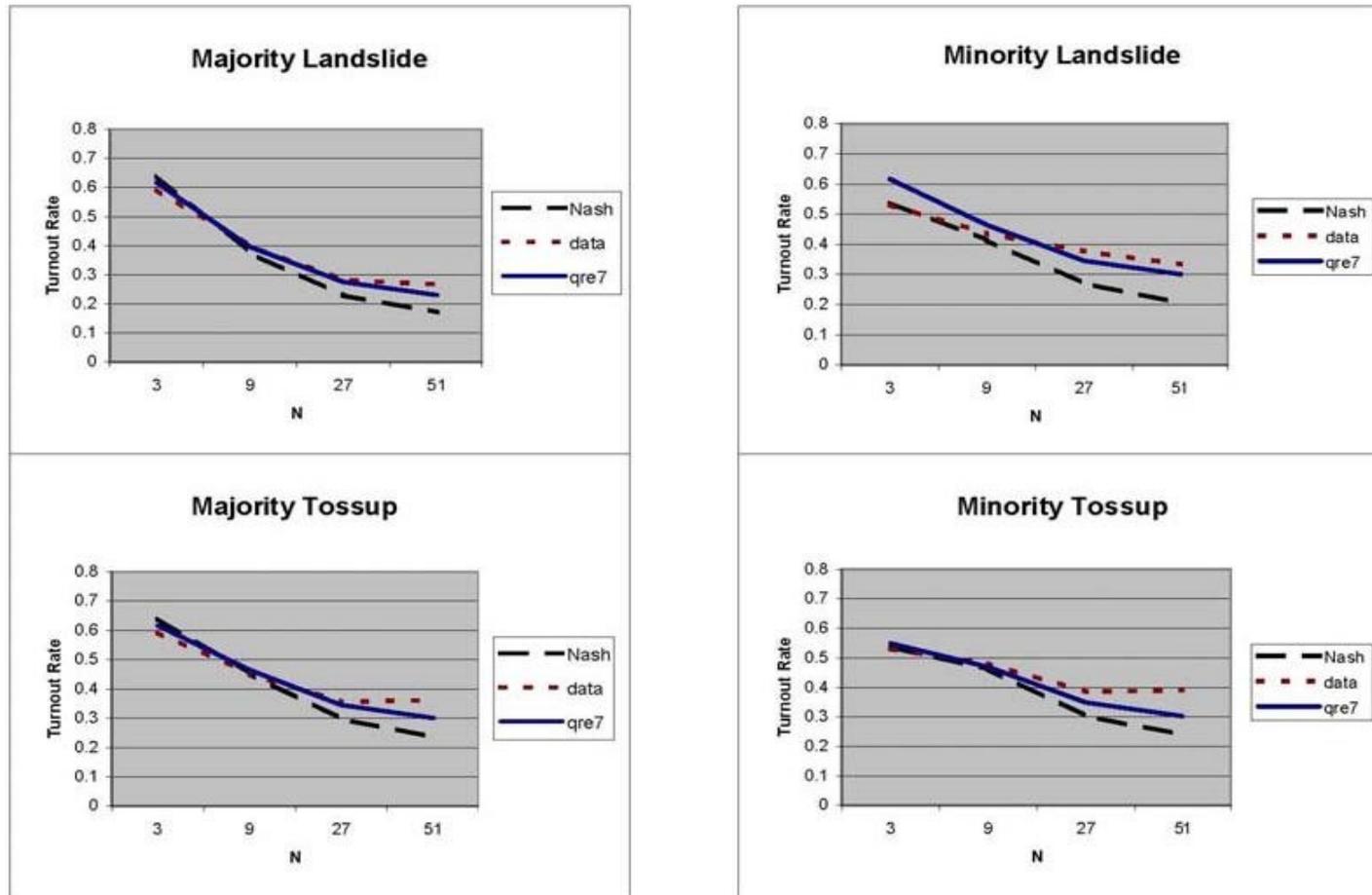
$N=3$  sessions conducted slightly differently  
only 50 rounds, since toss-up and landslide are identical  
sessions were conducted with either 12 or 15 subjects  
randomly rematched each round into subgroups of 3 each period,  
before being assigned a party and a voting cost  
allowed us to obtain a comparable number of subjects for the  $N=3$   
treatment as the other treatments.

## *Aggregate results - Turnout Rates*

**Table 3:** Turnout Rates - Comparison of Theory and Data

$N$	$N_A$	$N_B$	$\hat{P}_A$	$P_A^{Nash}$	$P_A^{\lambda=7}$	$\hat{P}_B$	$P_B^{Nash}$	$P_B^{\lambda=7}$
3	1	2	.530 (.017)	.537	.549	.593 (.012)	.640	.616
9	3	6	.436 (.013)	.413	.421	.398 (.009)	.374	.395
9	4	5	.479 (.012)	.460	.468	.451 (.010)	.452	.463
27	9	18	.377 (.011)	.270	.297	.282 (.007)	.228	.275
27	13	14	.385 (.009)	.302	.348	.356 (.009)	.297	.345
51	17	34	.333 (.011)	.206	.245	.266 (.008)	.171	.230
51	25	26	.390 (.010)	.238	.301	.362 (.009)	.235	.300

**Figure 5.1: Minority and Majority Party Turnout Rates As Function of Electorate Size**



- competition effect ( $H2$ ), underdog effect ( $H3$ ), and 3-voter counterexample to the underdog effect ( $H4$ ) supported without exception: all pairwise comparisons of turnout rates for those hypotheses are the same as predicted by the theory.
- noteworthy and surprising is support for  $H4$ : the minority turns out less than the majority in only one instance, which is precisely the one predicted by the theory ( $N_A=1, N_B=2$ )
- size effect,  $H1$ , supported except for comparison of turnout in tossup races with 27 versus 51 voters (quantitatively small and small relative to sampling error)
- NOTE: no parameters are estimated AT ALL

- turnout rates for the smallest electorate (N=3) *lower* than predicted by Nash
- turnout rates for the largest electorates (N=27,51) are *higher* than predicted by Nash
- in general turnout rates are closer to .5 than theory predicts
- mirrors findings by Goeree and Holt (2005) for a broad class of games

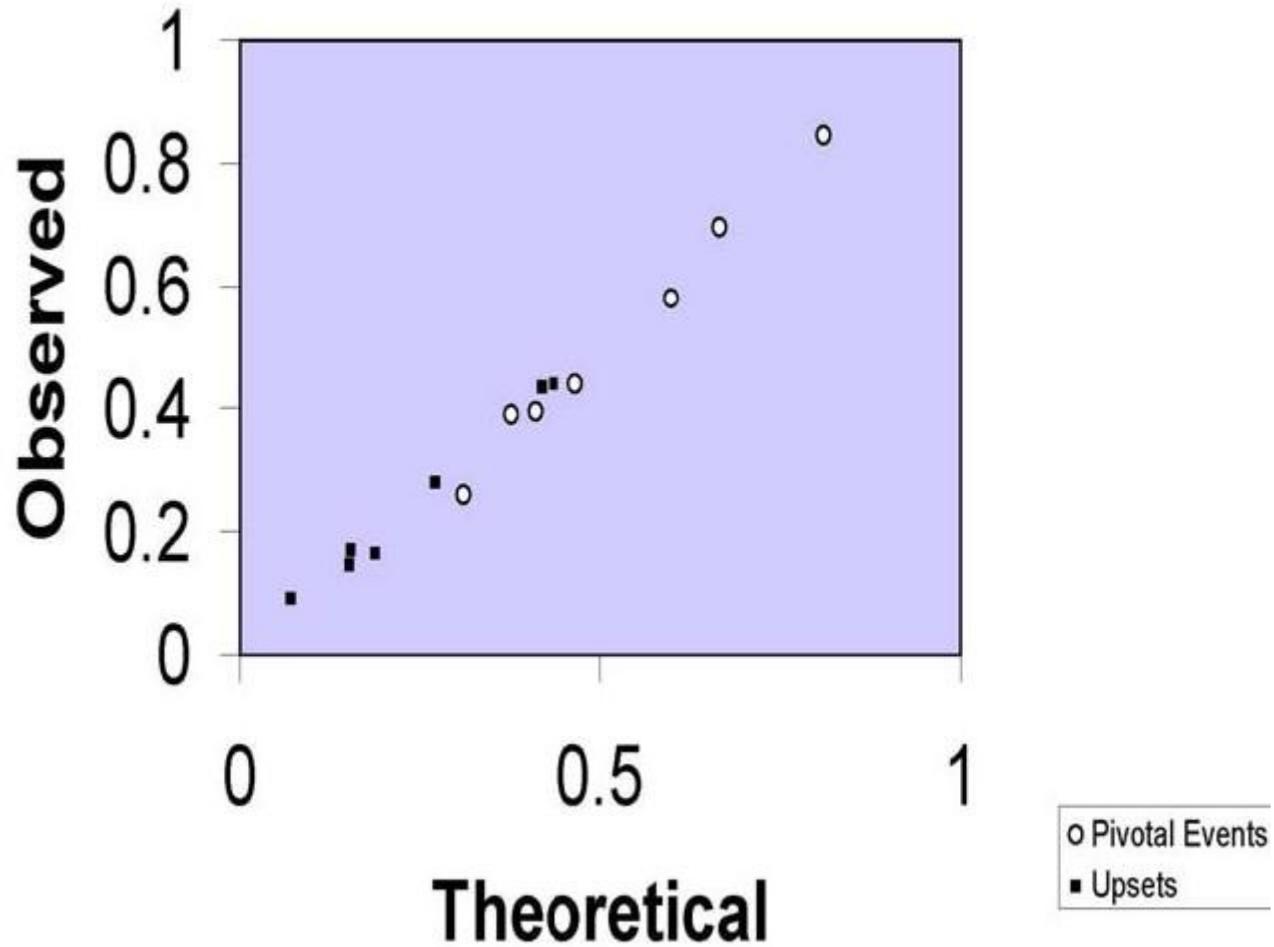
## *Framing*

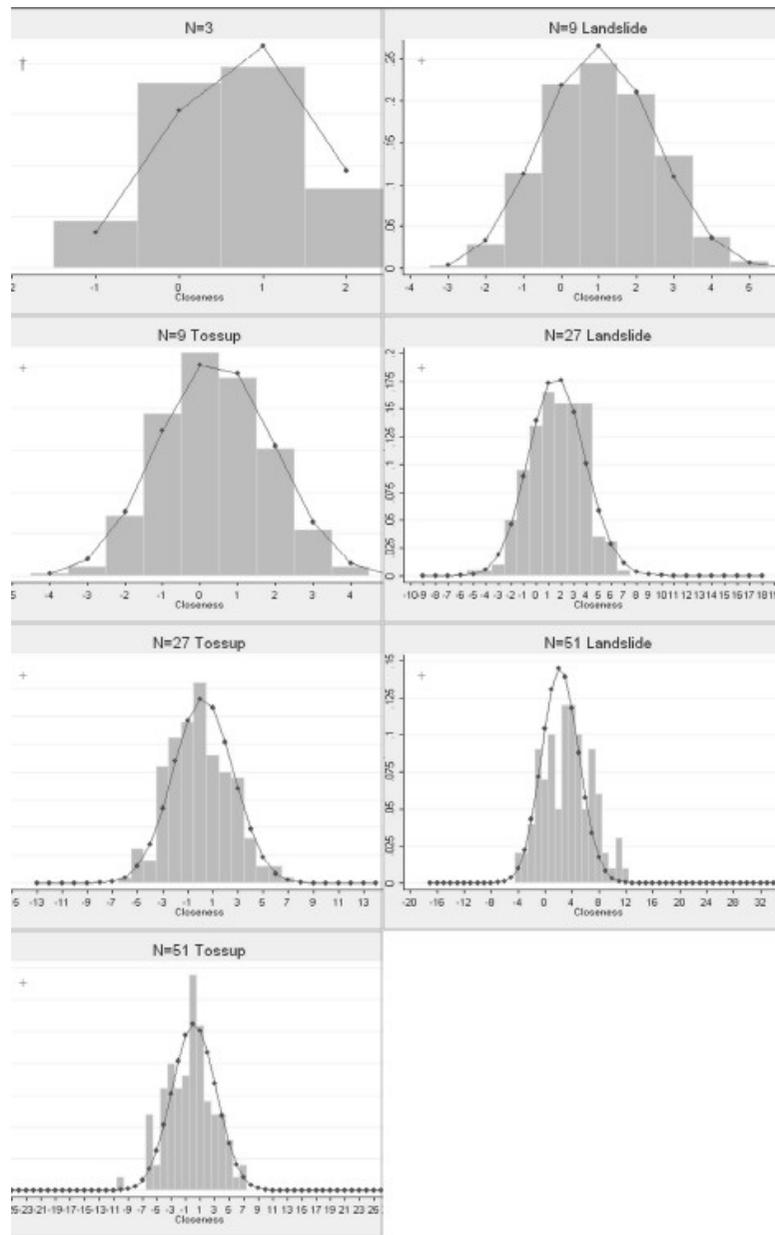
two additional sessions with  $N=9$ , as robustness check  
changed ``X" and ``Y" to ``Vote" and ``Abstain."  
round called "election"  
groups called "parties"

**Table 4:** Comparison of Turnout Rates Across Protocols  $N=9$

	Abstract Protocol	Election Context Protocol
$\hat{p}_A$ tossup	479 (.012)	0.475 (.022)
$\hat{p}_B$ tossup	.451 (.010)	0.480 (.018)
$\hat{p}_A$ landslide	.436 (.013)	0.460 (.025)
$\hat{p}_B$ landslide	.398 (.0094)	0.400 (.018)

*Aggregate results Upsets and Pivotal Events*





## ***Quantal Response Equilibrium turnout rates***

voter's turnout probability with voting cost is  $c$

$$\tau_j(c; \lambda) = \frac{1}{1 + e^{\lambda(c - \pi_j)}}$$

where  $\lambda$  is the “logit response parameter”

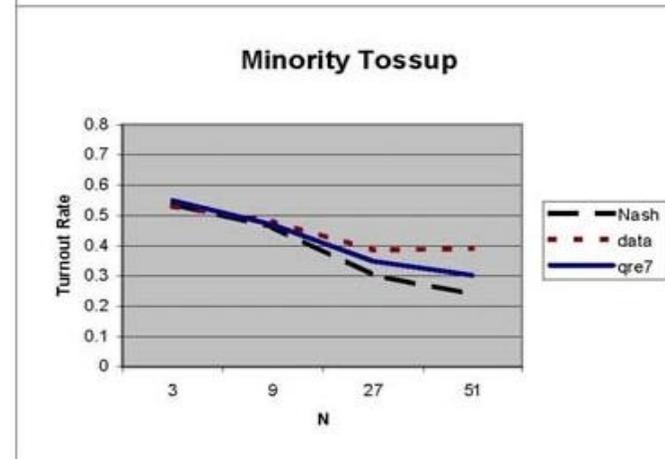
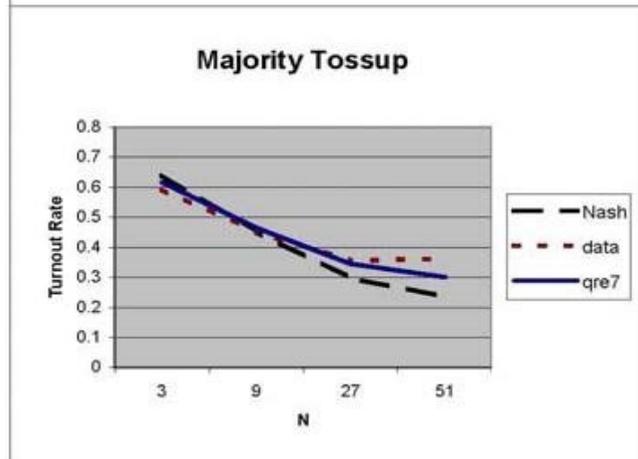
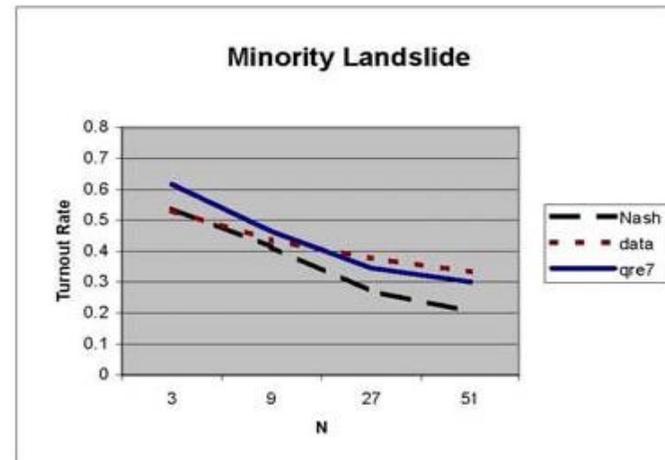
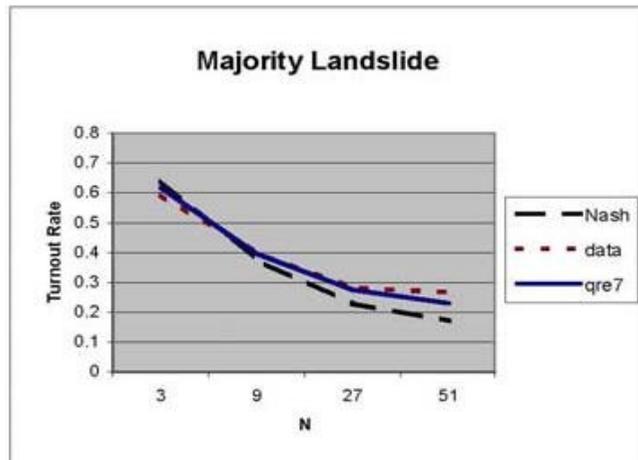
voter's ex ante turnout probability

$$p_j^*(\lambda) = \int_{-\infty}^{\infty} \tau_j(c; \lambda) f(c) dc$$

logit response parameter,  $\lambda$ , a free parameter

estimate a single value of  $\lambda$  using MLE over all treatments

maximum likelihood estimate is  $\lambda = 7$



## *turnout rates by “normalized” voting cost*

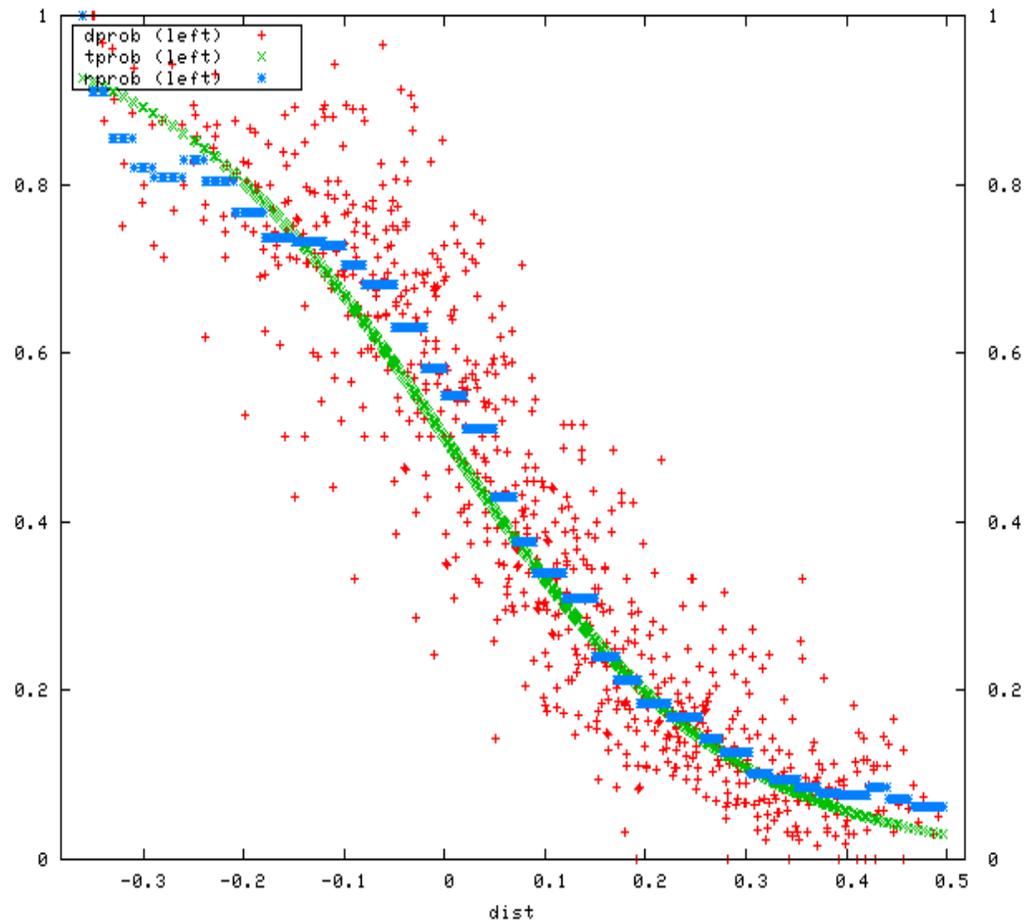
difference between a voter's actual voting cost and the logit equilibrium cutpoint (for  $\lambda = 7$ )

for example, if QRE cutpoint (indifference point) for an A voter in some treatment were, say, 15, and their actual cost were 25, their normalized voting cost would be 10

decreasing step-function curve averages the data across normalized cost intervals of .03.

HERE is big difference with Nash – at the individual level

**Figure 5.3:** Turnout Rates as Function of Cost, Compared to Logit Response with  $\lambda=7$



## Asymptotics

Nash: as number of voters goes to infinity, participation goes to zero

$$\text{QRE: participation rate } p_j^*(\lambda) = \int_0^{.55} \frac{1}{1 + e^{\lambda(c - \pi_j)}} \frac{1}{.55} dc$$

asymptotically probability of being pivotal  $\pi_j \rightarrow 0$

TABLE 10. Large Population Participation Rates

$\lambda$	0	1	2	3	4	5	6	7	28
$p$	0.50	0.43	0.37	0.31	0.27	0.23	0.20	0.17	0.05

(and this is for ridiculously high voting costs – expected cost  $\frac{1}{4}$  value of being dictator)